Heat flux prediction for hypersonic flows using a stabilized formulation

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Abstract

This work focuses on the heat flux prediction in hypersonic flow regimes using the finite-element based Streamline-Upwind
Petrov–Galerkin formulation enhanced with a discontinuity-capturing operator and weak enforcement of the Dirichlet boundary condition. The numerical formulation is validated on several benchmark cases including Mach 14 compression corner at 15° and 24°, 2D Mach 17 cylinder and 3D Mars Pathfinder re-entry vehicle at Mach 14. The numerical results are in very good agreement with the experiments or data available in the literature, showing the robustness of the numerical framework. Moreover, the newly proposed weak imposition of the no-slip boundary condition at the surface shows great potential for near-wall modeling of high-speed compressible flows.

Keywords Hypersonic flows · Stabilized methods · Heat transfer · Finite elements · Weak boundary conditions

1 Introduction

One of the main challenges faced by engineers during the design process of a hypersonic aircraft is the severe aerodynamic heating it experiences throughout its mission [40]. In the hypersonic regime, the interaction of air friction and complex flow phenomena, such as shock–shock and shock wave-boundary layer interactions, generates a significant amount of heat and conventional methods to provide a single-point temperature and heat flux values is insufficient. In recent years, the progress has been made in advancing numerical methods for aerothermal analysis of hypersonic flows [12, 29–32] and the accurate heat flux prediction was identified as one of the key issues [25]. In [23–25] the authors conducted an assessment of the classic finite volumes techniques, showing the sensitivity of these methods to the mesh topology and inviscid flux functions in predicting heat transfer. The finite element based methods, on the other hand, do not exhibit these behaviour and can be considered as a

promising alternative for hypersonic flows modeling [7, 8]. While both methods can handle very complex geometries [43], the finite elements allows a simplified implementation of boundary conditions and higher-order accuracy, which are nontrivial challenges in the context of finite volume methods [11].

Significant progress has been made over the past decades in the stabilized methods based on the Streamline-Upwind Petrov–Galerkin (SUPG) formulation [5, 17, 44, 45] enhanced with a discontinuity capturing (DC) operator [18, 27, 46–48, 52]. These methods have been successfully applied to various compressible flow problems with traditional linear finite elements and in a context of isogeometric analysis (IGA) with non-uniform rational B-splines (NURBS). This includes simulations of a delta-wing [49, 50], aircrafts and rotorcrafts [34, 35, 50, 51, 55], missiles [39], gas turbines [4, 26, 54], spacecraft parachutes [21, 41], re-entry vehicles in hypersonic regime [9] and more recently the hypersonic flows in thermo-chemical non-equilibrium [10, 22].

The selection of the stabilization parameters, including the element length calculation, estimation of the constants, plays a crucial role and require a careful balance between accuracy and stability [13]. The review of the stabilization parameters is outside of the scope of this article and the interested reader is referred to a review article on the stabilized methods for compressible flows in [19] and recent paper on stabilization and discontinuity-capturing parameters in [42].

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The current work extends the formulation presented in [9] for hypersonic flows by introducing the weakly enforced Dirichlet boundary conditions [4, 54]. The focus of this study is to examine the performance of weakly imposed essential boundary conditions in predicting heat flux in the hypersonic regime. The rest of the article is structured as follows. The mathematical formulation is described in Sect. 2, followed by the numerical tests presented in Sect. 3. The conclusion and future work are discussed in Sect. 4.

2 Numerical formulation

The set of compressible Navier–Stokes equations in conservative form can be written as

$$\partial_t \mathbf{U} + \nabla \cdot \mathbf{F} + \mathbf{S} = \mathbf{0}, \tag{1}$$

where ∂_t represents the Eulerian time derivative and the vector of the conservative variables $\mathbf{U}$, source term $\mathbf{S}$, flux vector $\mathbf{F}$, which consists of convective flux $\mathbf{F}^{conv}$, pressure flux $\mathbf{F}^p$ and diffusive flux $\mathbf{F}^{diff}$, are given by

$$\mathbf{U} = \begin{vmatrix} \rho \\ \rho\mathbf{u} \\ \rho\left(e + \mathbf{u}\cdot\mathbf{u}/2\right) \end{vmatrix} \quad \mathbf{F}^{conv} = \mathbf{U}\otimes\mathbf{u} \quad \mathbf{F}_p = \begin{vmatrix} \mathbf{0}^T \\ p\mathbf{I} \\ \mathbf{u}^T p \end{vmatrix}$$

$$\mathbf{F}^{diff} = -\begin{vmatrix} \mathbf{0}^T \\ \boldsymbol{\tau} \\ \mathbf{u}^T\boldsymbol{\tau} + \kappa\left(\nabla T\right)^T \end{vmatrix} \quad \mathbf{S} = \begin{vmatrix} 0 \\ \rho\mathbf{b} \\ \rho\left(\mathbf{b}\cdot\mathbf{u} + r\right) \end{vmatrix} \tag{2}$$

In Eq. 2, ρ is the density, $\mathbf{u}$ is the velocity vector, T is the thermodynamic temperature, p is the pressure, $\mathbf{b}$ is a body force, r is a heat source/sink, $e = c_v T$ is the internal energy, c_v is the specific heat at constant volume, κ is the thermal conductivity, $\mathbf{I}$ is the identity tensor, and $\boldsymbol{\tau}$ is the second order viscous stress tensor defined as

$$\boldsymbol{\tau} = \lambda\nabla\cdot\mathbf{u} + \mu\left((\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\right), \tag{3}$$

where λ is the bulk viscosity and μ is the dynamic viscosity. We also assume the equation of state for the ideal gas.

Together with Eq. 1, the Dirichlet boundary conditions $\mathbf{U} = \mathbf{U_G}$ on a domain boundary Γ_g are considered, where $\mathbf{U_G} = [\rho,\ \rho\mathbf{g}^T,\ \rho c_v\left(T_w + \mathbf{g}\cdot\mathbf{g}/2\right)]^T$, with $\mathbf{g}$ being the prescribed velocity vector, and T_w being the prescribed wall temperature.

Equation 1 can be also written in a quasi-linear form using a set of pressure-primitive variables $\mathbf{Y} = [p,\ \mathbf{u}^T,\ T]^T$ as

$$\mathbf{A}_0\,\partial_t\mathbf{Y} + \mathbf{A}\,\nabla\cdot\mathbf{Y} = \mathbf{0}, \tag{4}$$

where $\mathbf{A}_0 = \nabla_\mathbf{Y}\mathbf{U}$ is a transformation matrix from conservative to pressure-primitive variables, and the Jacobian matrix

$\mathbf{A} = \mathbf{A}^{conv} + \mathbf{A}^p + \mathbf{A}^{diff} = \nabla_\mathbf{U}\mathbf{F}\mathbf{A}_0$. In Eq. 4, the source term $\mathbf{S}$ is assumed to be zero. In this form, the Dirichlet boundary conditions become $\mathbf{Y} = \mathbf{Y_G}$, where $\mathbf{Y_G} = [p,\ \mathbf{u}^T,\ T]^T$. The reader is referred to the appendix of [9] for the explicit expressions of the matrices appearing in the quasi-linear form, and [14] for comparison of different set of variables.

The semi-discrete variational form is obtained by applying the method of weighted residuals to Eq. 4 with the integration by parts performed on the pressure and diffusive fluxes. To stabilize the formulation the SUPG term is added on the interior of each element [9, 54]. Moreover, due to the presence of strong shock waves in the flow, additional dissipation is added through the discontinuity capturing (DC) operator [1, 4, 9, 10, 27, 28, 54]. The final semi-discrete weak form can be written as

$$\int_\Omega \mathbf{W}^h \cdot \left(\mathbf{A}_0\cdot\mathbf{Y}^h_{,t} + \mathbf{A}^{conv}_i\cdot\mathbf{Y}^h_{,i}\right)d\Omega$$

$$-\int_\Omega \mathbf{W}^h_{,i}\cdot\left(\mathbf{A}^p_i\cdot\mathbf{Y}^h - \mathbf{K}_{ij}\cdot\mathbf{Y}^h_{,j}\right)d\Omega$$

$$+\sum_e^{N_{el}}\int_{\Omega_e}\mathbf{W}^h_{,i}\mathbf{A}^T_i\boldsymbol{\tau}_{SUPG}\mathbf{Res}\,d\Omega$$

$$+\sum_e^{N_{el}}\int_{\Omega_e}\mathbf{W}^h_{,i}\mathbf{K}_{DC}\mathbf{Y}^h_i\,d\Omega$$

$$+\int_{\Gamma_h}\mathbf{W}^h\cdot\left(\mathbf{A}^p_i\cdot\mathbf{Y}^h - \mathbf{K}_{ij}\cdot\mathbf{Y}^h_{,j}\right)n_i\,d\Gamma$$

$$\int_{\Gamma_g}\mathbf{W}^h\cdot\left(\mathbf{A}^p_i\cdot\mathbf{Y}^h\right)n_i\,d\Gamma - \int_{\Gamma_g}\mathbf{W}^h\cdot\left(\mathbf{K}_{ij}\cdot\mathbf{Y}^h_{,j}\right)n_i\,d\Gamma$$

$$-\int_{\Gamma_g}\left(\mathbf{K}_{ij}\cdot\mathbf{W}^h_{,j}\right)\cdot\left(\mathbf{Y}^h - \mathbf{Y}^h_\mathbf{G}\right)n_i\,d\Gamma$$

$$+\int_{\Gamma_g}\mathbf{W}^h\cdot\boldsymbol{\tau}_B\cdot\left(\mathbf{Y}^h - \mathbf{Y}^h_\mathbf{G}\right)d\Gamma$$

$$-\int_{\Gamma_g}\mathbf{W}^h\cdot\{\hat{\mathbf{A}}_\mathbf{n}\}_-\cdot\left(\mathbf{U}(\mathbf{Y}^h) - \mathbf{U}(\mathbf{Y}^h_\mathbf{G})\right)d\Gamma$$

$$= 0, \tag{5}$$

where $i, j = 1\ldots n_d$ are the indexes for the spatial dimension, d is the number of space dimensions, $\Omega \subset \mathbb{R}^d$ is the volumetric domain which is descritized using linear finite elements as $\Omega = \cup_{e=1}^{n_{el}}\Omega_e$. The natural boundary conditions are enforced on $\Gamma_h \in \Gamma$ and the essential boundary conditions are enforced on $\Gamma_g \in \Gamma$. n_i is the ith component of outward unit surface normal vector $\mathbf{n}$. In Eq. 5, $\mathbf{Y}^h$ and $\mathbf{W}^h$ are the solution and test functions from the finite dimensional spaces $\mathbf{S}^h$ and $\mathbf{V}^h$ respectively. $\mathbf{Res}$ is the strong-form residual. $\boldsymbol{\tau}_{SUPG}$ is the stabilization matrix in pressure-based primitive variables [9]

$$\boldsymbol{\tau}_{SUPG}$$

$$= \mathbf{A}_0^{-1} \left(\frac{4}{\Delta t^2} \mathbf{I} + G_{ij} \mathbf{A}_i \mathbf{A}_j + C_I \ G_{ij} G_{kl} \mathbf{K}_{ik} \mathbf{K}_{jl} \right)^{-\frac{1}{2}} \quad (6)$$

where Δt is the time step size, C_I is a positive constant derived from an appropriate element-wise inverse estimate ($C_I = 3$ for all cases presented in this work) and G_{ij} are the components of the element metric tensor. $\mathbf{K}_{DC}$ is a diagonal diffusivity matrix for discontinuity capturing

$$\mathbf{K}_{DC} = C_{DC} \left(\frac{\mathbf{Res}^{\mathrm{T}} \tilde{\mathbf{A}}_0^{-1} \mathbf{Res}}{G_{ij} \mathbf{U}_{,i}^{\mathrm{T}} \tilde{\mathbf{A}}_0^{-1} \mathbf{U}_{,j}} \right)^{\frac{1}{2}} \mathbf{A}_0, \quad (7)$$

where C_{DC} is a positive constant ($C_{DC} = 1$ for all cases presented in this work), and $\tilde{\mathbf{A}}_0^{-1}$ is the transformation from conservative to entropy variables [6, 14]. Last five terms in Eq. 5 originate from weak imposition of the Dirichlet boundary conditions. In Eq. 5, $\boldsymbol{\tau}_B$ is a diagonal positive semi-definite matrix of stabilization parameters [54] (in this work a positive constant used in the definition of $\boldsymbol{\tau}_B$ is 4), and $\{\mathbf{A_n}\}_- = \{\mathbf{A}_i n_i\}_- = \mathbf{MT}\{\boldsymbol{\Lambda_n}\}_- \mathbf{T^{-1}M^{-1}}$ [53]. Operator $\{\cdot\}_-$ isolates the part of a matrix-valued argument corresponding to the eigenvalues with negative real parts. $\boldsymbol{\Lambda_n}$ is the diagonal matrix of the eigenvalues of $\mathbf{A_n}$ and $\mathbf{T}$ is the transformation matrix constructed using the eigenvectors as columns [53]. $\mathbf{M}$ is the Jacobian matrix $\partial \mathbf{U}/\partial \tilde{\mathbf{U}}$, where $\tilde{\mathbf{U}} = [\rho, \ \mathbf{u}^T, \ p]^T$, and defined as

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 \\ 0 & 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & 0 & (\gamma-1)^{-1} \, e_{tot}/e \end{bmatrix}, \quad (8)$$

where γ is the adiabatic index and $e_{tot} = e + \mathbf{u} \cdot \mathbf{u}/2$ is the total energy.

Remark 1 We focus on the last term of Eq. 5 and consider inflow and wall boundary conditions. We set the inflow boundary conditions strongly on all the variables, which implies $\mathbf{W}^h = 0$ and $\mathbf{U}(\mathbf{Y}^h) = \mathbf{U}(\mathbf{Y}_G^h)$. As a result, this term does not contribute to the formulation. In the case of wall boundary conditions, we set the temperature strongly and velocity weakly. This results in $\mathbf{U}(\mathbf{Y}^h) - \mathbf{U}(\mathbf{Y}_G^h) = [0, \ \rho \, (\mathbf{u} - \mathbf{g}), \ \rho \, (\mathbf{u} \cdot \mathbf{u}/2 - \mathbf{g} \cdot \mathbf{g}/2)]^T$. In addition, $W_5{}^h = 0$. Because $\{\hat{\mathbf{A}}_\mathbf{n}\}_-$ is a fully populated matrix, the weak boundary condition terms will produce a contribution that has a mismatch in the momentum and kinetic energy density.

The semi-discrete formulation defined in Eq. 5 is integrated in time using the generalized-α method [3, 9, 20, 54] (in this work the generalized-α parameter $\rho_\infty = 0.5$). All simulations are done with a constant time step. The Newton–Raphson method is used to solve the resulting non-linear

system of equations. At each non-linear iteration, the linear system is solved through the Generalized Minimal Residual method (GMRES) [37]. The convergence criteria used for non-linear and linear solvers is the three orders of magnitude reduction in the norm of the residual.

3 Numerical results

In this section the numerical results for the compression corner, the cylinder and the Mars Pathfinder re-entry vehicle are presented. The results are compared with available experimental data and if no experimental data is available the results are compared with numerical tests from the literature.

3.1 Compression corner

The compression corner in hypersonic flows belongs to a category of cases associated with shock wave-boundary layer interaction (SWBLI) problems [2]. It is characterized by a flat plate section followed by an inclined section at a certain angle α. The flow physics is characterized by a primary shock wave downstream the corner and a separation region, the size of which varies depending on the Mach number and the angle α. The primary shock originates from the streamlines deflection due to the wedge angle α. The strong adverse pressure gradient due to the primary shock wave compression cause the flow to separate around the corner. A secondary shock wave forms because of the flow separation. The strength of the separation shock also depends on the Mach number and the angle α. The separation shock strengthens the primary shock, resulting in higher pressure and wall heat flux. The impingement of the primary shock on the boundary layer on the inclined section of the model creates a high pressure region that compresses the boundary layer, making it thinner, which generates high surface heat flux. Two compression corner cases are considered, one at $\alpha = 15°$ and the other one at $\alpha = 24°$. The flow for the compression corner at $\alpha = 15°$ remains attached and it does not separate at the corner, while a wide separation region is observed for the case at $\alpha = 24°$.

3.1.1 Compression corner at 15°

The compression corner at $\alpha = 15°$ is presented first. The free-stream is characterized by a Mach number $M_\infty = 14.1$ and a Reynolds number $Re_\infty = 2.6 \times 10^5$ [1/m]. At the inlet, $p_\infty = 10.02$ [Pa], $T_\infty = 72.22$ [K] and $u_\infty = 2401.9253$ [m/s] are imposed strongly. At the wall, the no-slip boundary conditions are imposed weakly, while the isothermal condition $T = 297.22$ [K] is only imposed strongly. At the outflow, as demonstrated in [38], the boundary integral is included for all flux terms, including the viscous and heat transfer contributions. The temperature
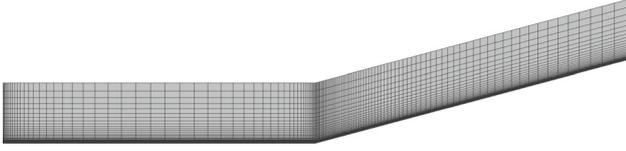
Fig. 1 Coarse computational grid for the compression corner at $\alpha = 15°$
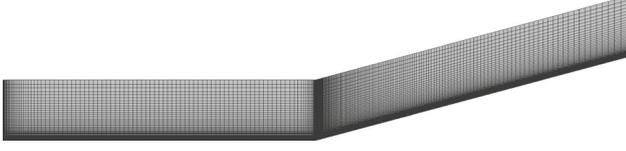
Fig. 2 Medium computational grid for the compression corner at $\alpha = 15°$
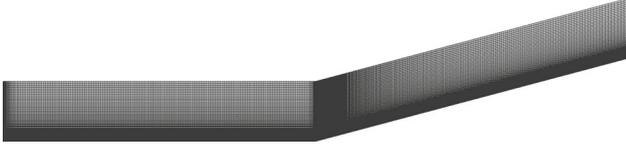
Fig. 3 Fine computational grid for the compression corner at $\alpha = 15°$

dependent viscosity is computed through the Sutherland law defined as:

$$\mu = \mu_{ref} \cdot \frac{T^{1.5}}{T + S}, \qquad (9)$$

where $\mu_{ref} = 1.458 \times 10^{-6}$ [Pa·s] and $S = 110.33$ [K]. The thermal conductivity is obtained from the Prandlt number definition as follows:

$$\lambda = \mu C_p / Pr, \qquad (10)$$

where $Pr = 0.72$ and the specific heat at constant pressure is constant and given as a function of the specific heat ratio $\gamma = 1.4$ and the universal gas constant R, as $C_p = \frac{\gamma R}{\gamma - 1}$.

Three computational structured grids are considered for this case. The elements are clustered close to the leading edge of the model and close to the wall. The coarse grid has 125 elements in the streamwise direction and 35 elements in the normal to the surface direction and is shown in Fig. 1. The medium grid is 250 by 70 elements and is shown in Fig. 2. The fine grid is 500 by 140 elements in the streamwise and wall normal directions respectively and is shown in Fig. 3. The grid convergence study is performed on non-dimensional coefficient for the surface heat flux defined as follows:

$$C_h = \frac{q}{\rho_\infty u_\infty (H_\infty - H_{wall})}, \qquad (11)$$

where H_∞ and H_{wall} is the total enthalpy at the inflow and the wall respectively. The simulations were done with a constant time step of 1×10^{-6} [s] until a steady-state solution is
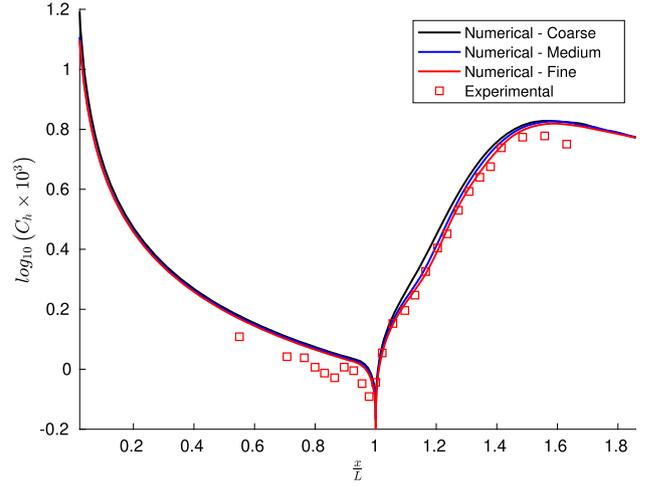
Fig. 4 Distribution of the surface heat flux coefficient C_h for the compression corner at $\alpha = 15°$
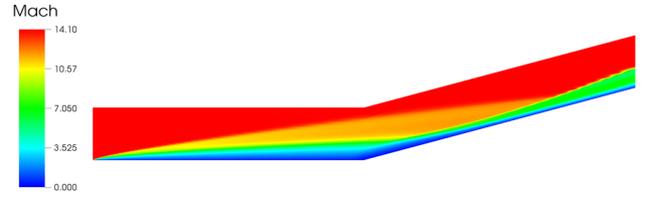
Fig. 5 Mach number contours for the compression corner at $\alpha = 15°$

obtained. The distribution of the surface heat transfer coefficient is illustrated in Fig. 4. The results agree quite well with the experiments data obtained by Holden [15] with slight overprediction of the heat flux in the flat plate section, but excellent agreement is obtained at the region where the boundary layer is thinned by the compression generated by the primary shock. Based on this study the medium mesh is selected to vizualize the flow topology and also study the 24° compression corner. The flow topology is shown in Fig. 5. No flow separation is observed as discussed earlier and smooth solution is obtained throughout the domain, showing the accuracy of the stabilization parameters and the discontinuity capturing term.

3.1.2 Compression corner at 24°

Next, the compression corner at $\alpha = 24°$ is investigated. The boundary conditions and fluid properties are similar to 15° case. As suggested by [36], the three-dimensional effects cannot be neglected and they strongly affect the prediction of the extent of the separation region. The medium computational grid from 15° case is used with 25 elements in the transverse direction and is shown in Fig. 6. The minimum element size at the wall-normal direction is 5×10^{-5} [m] and a constant time step of 1×10^{-6} [s] is used.
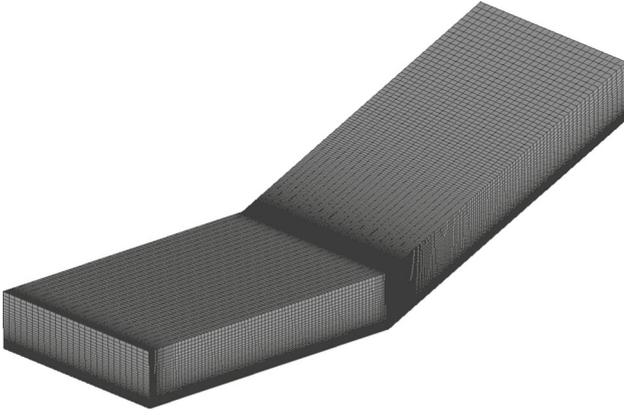
Fig. 6 Medium computational grid for the compression corner at $\alpha = 24°$
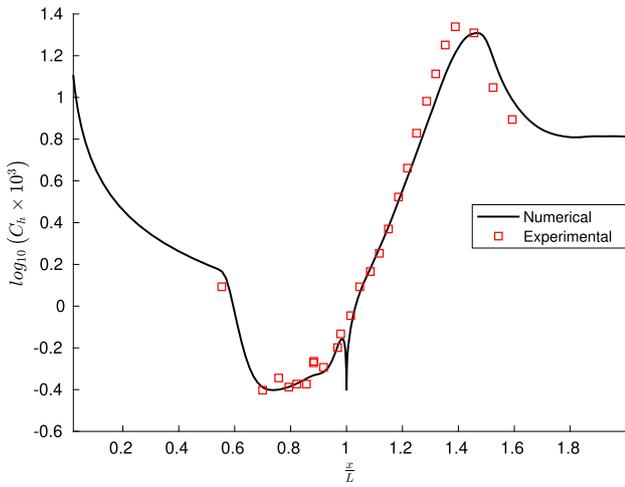
Fig. 7 Distribution of the surface heat flux coefficient C_h for the compression corner at $\alpha = 24°$
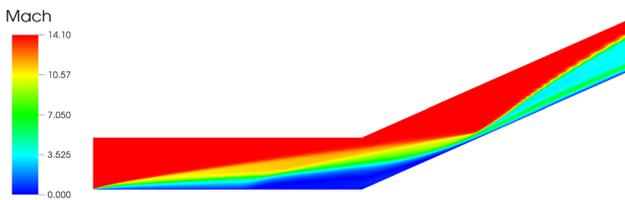
Fig. 8 Mach number contours for the compression corner at $\alpha = 24°$

The heat flux distribution is shown in Fig. 7 and in a good agreement with the experimental data [15], especially in the separation region. Only the slight underprediction of the peak heat flux can be observed further downstream. The flow topology can be observed in Fig. 8, showing a wide separation region around the corner of the model. The numerical solution accurately capture the impingment of the separation shock on the primary shock and the thinning of the boundary layer due to the high pressure.
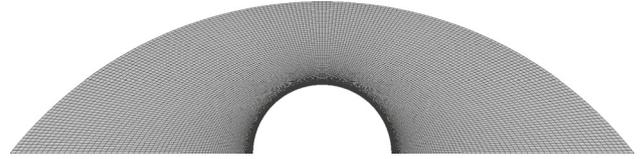
Fig. 9 Computational grid for the cylinder case

3.2 Flow over a 2D cylinder

In this section the laminar cylinder case immersed in a hypersonic flow is presented. This is a standard 2D benchmark case for hypersonic flows. The free-stream is defined by a Mach number $M_\infty = 17$ and a Reynolds number $Re_\infty = 7.52 \times 10^5$. The free-stream conditions at the inlet $p_\infty = 57.4\,[\text{Pa}]$, $T_\infty = 200\,[\text{K}]$, $u_\infty = 5000\,[\text{m/s}]$ are imposed strongly. The no-slip boundary condition is imposed weakly at the wall, while the isothermal condition $T_w = 500\,[\text{K}]$ is imposed strongly. At the outflow, as demonstrated in [38], the boundary integral is included for all flux terms, including the viscous and heat transfer contributions. The temperature dependent viscosity is calculated using the Sutherland law as in Eq. 9 and the thermal conductivity as in Eq. 10. For this case a structured computational grid is used and is shown in Fig. 9. The grid is characterized by 180 elements in the azimuthal direction and 140 elements in the radial direction. The elements are clustered towards the wall, where the smallest element size in a radial direction is $5 \times 10^{-5}\,[\text{m}]$. A constant time-step of $1 \times 10^{-6}\,[\text{s}]$ is used.

The flow field developed around the 1 [m] radius cylinder, is shown in Fig. 10. The bow shock developed around the cylinder, shows some oscillations towards the outlet boundary. These small oscillations are grid related, since the shock-grid alignment is progressively lost moving away from the stagnation region.

The distribution of the surface heat flux is shown in Fig. 11. The results obtained by the present formulation are compared to the numerical data obtained using LAURA code by Quattrochi [33]. The results slightly underpredict the data from Quattrochi in the region between the stagnation point and the bottom part of the cylinder, while the peak heat transfer
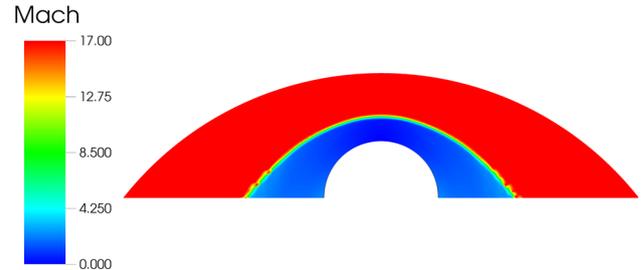
Fig. 10 Mach number contours for the cylinder case
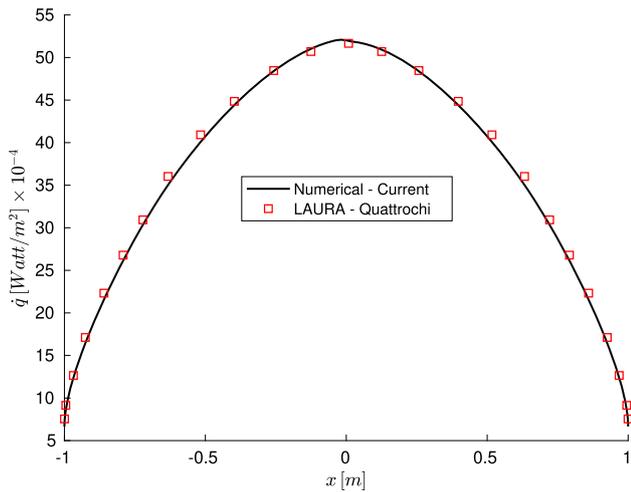
Fig. 11 Distribution of the surface heat flux for the cylinder case

Fig. 13 Cross-sectional slice of the Mach number contours for the Mars Pathfinder re-entry vehicle simulation

3.3 Mars Pathfinder re-entry vehicle

In this section the 3D Mars Pathfinder re-entry vehicle is analysed. A detailed numerical and experimental aerothermodynamics analysis of the re-entry vehicle in hypersonic regime, have been carried out in [16]. The geometry of this case is taken from the experimental model defined as $MP - 1$ in [16]. The computational grid is shown in Fig. 12. The grid is unstructured and contains a refinement section around the capsule. The resolution of the boundary layer grid is shown in Fig. 12b and it is characterized by 27 layers with the smallest element size in the wall-normal direction of 1.25×10^{-5} [m]. The free-stream is characterized by a Mach number $M_\infty = 10$ and a Reynolds number $Re_\infty = 3.187 \times 10^6$. The free-stream conditions imposed at the inlet strongly are $p_\infty = 130.5$ [Pa], $u_\infty = 1415$ [m/s], $T_\infty = 51.63$ [K]. The wall is isothermal at $T = 300$ [K], which is imposed strongly, and the no-slip boundary con-

predicted by the present formulation agrees quite well with the one obtained in [33].

dition is imposed weakly. At the outflow, as demonstrated in [38], the boundary integral is included for all flux terms, including the viscous and heat transfer contributions. The flow topology can be seen in Fig. 13. The flow field is mainly characterized by a bow shock in front of the body. Behind the vehicle two symmetric recirculation regions are observed. A shear layer develops between the post shock region and the recirculation region and an oblique shock wave develops downstream the vehicle on the sting.

The surface heat transfer distribution is shown in Fig. 14 as a function of the curvilinear coordinate S/R_b, where R_b is the vehicle radius. The results are compared to the experimental data obtained by Hollis for the Test 293 Run 007 [16]. The numerical results are in a good agreement with the experiment, mostly within the reported uncertainty, except for the location of the shock wave on the sting, where the results are slightly underpredicted. The slight overprediction of heat transfer in the front part of the vehicle, downstream the stagnation point, may be related to a coarser grid resolution at the shock location, with respect to the resolution right in front of the stagnation point.
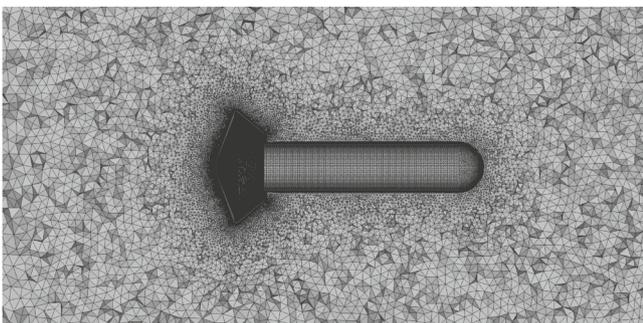
(a) Cross-sectional slice of the computational grid.
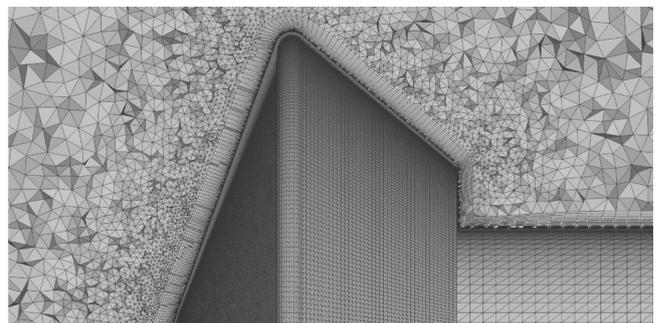
(b) Zoom on the boundary layer grid.

Fig. 12 Computational grid for the Mars Pathfinder re-entry vehicle
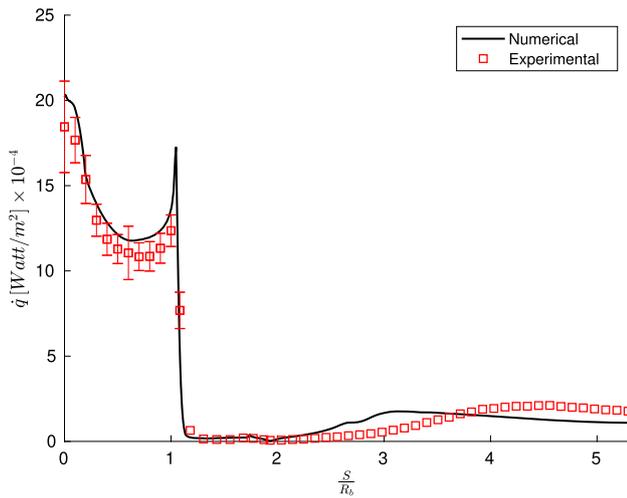
Fig. 14 Distribution of the surface heat transfer for the Mars Pathfinder re-entry vehicle simulation

4 Conclusion

The present work proposes the stabilized finite element formulation enhanced with a weak imposition of the no-slip boundary condition. The formulation is tested on the prediction of the surface heat flux for vaarious benchmark cases, including the compression corner at 15° and 24°, the 2D cylinder at Mach 17 and the 3D Mars Pathfinder re-entry vehicle. The numerical results are compared with the available experimental data and numerical data from in the literature. The overall good agreement is obtained showing the suitability of the stabilized formulation in predicting essential quantities such as the heat transfer in the context of hypersonic flows. The imposition of the no-slip boundary condition in a weak way provides a promising alternative to a classical imposition of the Dirichlet boundary condition. A further investigation, however, is needed. Particular attention should be given to the weak imposition of the isothermal condition at the wall. This will be the next step, where a full comparison between the strongly imposed wall boundary conditions and the weakly imposed boundary conditions will be carried out.

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