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Distributed Decision Propagation in Mobile-agent Proximity Networks★

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Abstract

This paper develops a distributed algorithm for decision/awareness propagation in mobile-agent networks. A time-dependent proximity network topology is adopted to represent a mobile-agent scenario. The agent-interaction policy formulated here is inspired from the recently developed language-measure-theory. Analytical results related to convergence of statistical moments of agent states are derived and then validated by numerical simulation. The results show that a single (user-defined) parameter in the agent interaction policy can be identified to control the tradeoff between *Propagation Radius* (i.e., how far a decision spreads from its source) and *Localization Gradient* (i.e., the extent to which the spatial variations may affect localization of the source) as well as the temporal convergence properties.

1. Introduction

Analysis and development of distributed decision propagation and control mechanisms in mobile-agent networks have drawn much attention due to their relevance in engineering problems. For example, surveillance and reconnaissance by autonomous vehicles with limited capabilities, trust establishments in mobile ad hoc networks (MANETs) [1] and threat monitoring by mobile sensor networks. In many applications, diffusion of aggregated information is more relevant compared to individual sensor information [2], [3] mostly due to its robustness to individual agent's failure in detection/communication. Furthermore, in a resource-constrained environment, mobile agents have potential advantages over static networks in terms of coverage and time-criticality. In this context, this paper deals with global propagation of a localized awareness in a leaderless environment in a robust and completely distributed manner.

In general, there are two aspects of interacting agent systems, namely (i) network topology and (ii) agent interaction dynamics. Network topology is inherently time-varying in the present context, which makes the analysis of such complex systems much harder compared to their static counterparts. Usually, similar time-varying situations arise in social networks [4] and they are modeled by various graphical structures, such as: multiple instances of uniform random graphs, scale-free networks and small-world networks [5]. Synchronization problems have been solved for time-varying networks where essentially the network topology is modeled as fast switching among a finite number of instances of random graphs with same specifications [6]. However, all such models do not necessarily consider the agent mobility statistics or inter-agent communications due to proximity. Recently, so-called proximity networks [7] (also called the moving neighborhood networks [8]) has been analyzed to model contact/collision-based disease spreading. This may be considered as the first step towards analyzing the mobile-agent scenario in an actual sense. In a recent paper [9], the current authors used such developments to model mobile-agent networks for engineering applications. The mobile-agent network used in this paper follows the same structure. Regarding the second aspect of the problem, distributed agent interaction dynamics for decision propagation has several mechanisms available in literature, examples are game theoretic [1], biology inspired, physics inspired (Ising/Potts models) [10], bootstrap percolation [11] and majority voting [12]. Gossip algorithms are the most studied interaction dynamics in the context of consensus [13]. However, in many applications, large groups of agents do not seek consensus. Often localized percolation of decision is desired to localize the information source.

The main contribution of this paper is the development of a distributed decision propagation algorithm inspired from the recently developed *language-measure theory* [14][15] for a time-dependent network topology. A single parameter in the

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algorithm is identified to control the tradeoff between *propagation radius* (i.e., how far a decision spreads from its source) and *localization gradient* (i.e., the extent to which the spatial variations may affect localization of the source). Analysis of (up to second order) moment dynamics [16], [17] is presented and the results are validated by numerical simulation. Variance analysis is performed under the following two conditions:

(i) *Congruous time-scale:* when the evolution of network topology and dynamics of agent interactions have similar time scales. and

(ii) *Disparate time-scale:* when faster dynamics of agent interactions can be treated as singular perturbations with respect to the slower evolution of network topology.

The paper is organized in six sections including the present one. The representation of a mobile-agent scenario in terms of proximity networks and the agent interaction policy are presented in Section 2. Section 3 presents the main results including their physical significance. These results are validated by numerical simulation in Section 4. Finally, the paper is summarized and concluded in Section 5 with recommendations for future work. Two appendices are provided to (i) explain the proximity networks in a greater detail and (ii) to briefly describe the basic concepts of the language-measure theory.

2. Formulation of the Problem

Let us consider the case of multiple agents performing surveillance, where the agents are tasked with detection of threats in a given region. A typical example of such a threat could be plumes of harmful chemicals that have to be detected. Taking into account the nature of these threats, they may be modeled as a local hotspot within the surveillance region. Only a few agents that search areas within the hotspot have a non-zero probability of detecting the threat. The aim of this paper is to develop a distributed and leader-less algorithm for mobile agents that is able to disseminate the information of a threat to other agents that may be far off from the local hotspot in a controlled fashion. Previous literature [18] have extensively studied the gradient-based approaches for detection of a hotspot. These approaches primarily focus on the moving agents towards the hotspot based on distributed estimation of gradients. However, in this application, it is required that all agents should become cognizant of the presence of the threat while operating and monitoring in their own respective local areas. In the proposed approach, the presence of a hotspot does not affect the motion of the agents. Instead, the information states of other agents are updated to reflect the required level of awareness that the agents should possess regarding the threat. The motivation here is to disseminate information away from the local hotspot to the entire population of agents. This section describes the setup of mobile-agent population in terms of proximity networks [7] and subsequently formulates the agent interaction policy.

A. Model Description

Proximity network is a particular formulation of time-varying mobile-agent networks, inspired from social networks. In this setup, mobile agents move around in an operational region with their own mobility characteristics. They communicate with each other as they become proximal to each other; a link is established from the network perspective between two communicating agents. The network links do not necessarily affect the mobility characteristics of the agents. Once a link is established, it is kept for a certain time period (which is termed as the message lifetime in this paper). As time progresses, more links are established and at the same time, older links disappear after expiry of their respective message lifetime. In this fashion, the network evolves in time. A specific scenario considered in the paper is formally presented in the sequel.

Let the area of a two dimensional (Euclidean) operational region be A. In the present case, A is assumed to be a square area with side length L, i.e., $A = L^2$. Initially, N agents are distributed randomly in the given area, and the agent density is defined as $\rho = N/A$. The uniform radius of communication for each agent is denoted by R, i.e., two agents can only communicate (e.g., to exchange messages) when the distance between them is less than R. The agents move in a 2-D random walk fashion where the speed v is same for all agents in the current setup. The random walk is realized by independently choosing a direction of motion from a uniform distribution $U(0, 2\pi)$, by all agents at each time step. During its motion, each agent broadcasts a message over a certain time window that is called the *message lifetime* L_m. In the present context, the message can be information-related to an agent's belief regarding its environment. At the same time, the agent receives similar messages from other proximal agents, which may come within the communication radius R. After expiry of a message lifetime, an agent possibly updates its belief based on its own observation and messages from other agents. This aspect is formally addressed in the next section.

In contrast to the faster time-scale (t) of agent motion, the algorithm for updating the agents' beliefs runs on a slower time-scale (denoted as τ). The time-scale for updating the belief is chosen to be slower as it allows for sufficient interactions among the agents, especially if the density of agents is low. After the updating, an agent starts broadcasting its new belief for another window of the message lifetime. For example, if the message lifetime L_m is very small, then the network may not be able to build up over time and possibly remains sparse. On the other hand, the network would eventually become fully connected as $L_m \to \infty$. Thus, to capture temporal effects in a realistic setting, L_m should be appropriately chosen based

on other network parameters. It is noted that, although updating of messages may occur in a non-synchronous manner in the agent population, only synchronous updating is considered in this paper for analytical tractability of the agent interaction policy without explicitly addressing the issue of obstacle avoidance. In this context, the notion of the degree of a network is introduced below.

Definition 2.1: (Degree of a node (or an agent)) The degree (d^i) of a node (i) is defined to be the number of distinct nodes in the network, to which it connects (e.g., for information communication) within a specified message lifetime L_m.

A brief discussion on the nature of the distribution of the degree of a node and the expected degree of this network class is provided in Appendix A while the details are reported in [9].

B. Agent Interaction Policy

The agent interaction policy developed in this paper is essentially inspired from the concepts of signed real measure of probabilistic regular languages generated by probabilistic finite state automata (PFSA) [14], [15]. However, the details are not presented here for simplicity and only the policy is described in a self-sufficient way. A brief discussion on the theory of language measure is provided in Appendix B.

The PFSA is developed on a graphical interaction model among the agents by the process described below.

1) Interaction Graph: The interaction graph is constructed in terms of the adjacency matrix of the mobile agent network after the expiry of the message lifetime L_m. To this end, the following definitions are introduced.

Definition 2.2: (Adjacency Matrix [19]). Let a time-dependent (in the slow-scale τ) graph be denoted as G. The adjacency matrix A of the graph G is defined such that its element a_{ij} in the ij^{th} position is unity if the agent i communicates with the agent j in the time period of L_m; otherwise the matrix element a_{ij} is zero. To eliminate self-loops, each diagonal element of the adjacency matrix is constrained to be zero.

The algorithm for simulating a proximity network in the current setting is provided below:

Algorithm 1: Proximity Network simulation

Initialize locations of N agents randomly in a 2-D region
τ_{end}: Total simulation time in the slow scale
$\tau = 1$
while $\tau < \tau_{end}$ **do**
  $a_{ij}|_\tau = 0$ for all i, j
  for $t = 1 \rightarrow L_m$ (Fast time scale of agent mobility) **do**
    Move each agent by one step with speed v in randomly chosen directions
    for all Agents i, j **do**
      if $dist(i, j) < R$ (Euclidean distance between agents i, j less than communication radius) **then**
        $a_{ij}|_\tau = 1$
      end if
    end for
  end for
  $\tau \leftarrow \tau + 1$
end while

Definition 2.3: (Laplacian Matrix [19]) The Laplacian matrix ($\mathcal{L}$) of a graph G is defined as:

$$\mathcal{L} = D - A$$

where the degree matrix D is a diagonal matrix with d^i as its i^{th} diagonal element, where d^i is the degree of the node i (see Definition 2.1).

Definition 2.4: (Interaction Matrix [19]) The agent interaction matrix Π is defined as:

$$\Pi = I - \beta\mathcal{L}$$

where the parameter β is chosen appropriately such that Π becomes a stochastic matrix and its second largest eigenvalue satisfies the condition $|\lambda_2(\Pi)| < 1$.

In the context of proximity networks, the requirement of keeping Π as a stochastic matrix in Definition 2.4 is achieved by setting $\beta = 1/(\bar{d} + 1)$, where $\bar{d}$ is a (positive integer) parameter that is pre-determined off-line. To satisfy this condition on-line, an agent ignores communications with distinct agents that are beyond the $\bar{d}$ agents within the message lifetime L_m. However, the expected degree distribution of the network is obtained off-line too at the design stage (see Appendix A); therefore, $\bar{d}$ is chosen to be large enough such that the probability that the degree $d^i > \bar{d}$ for any node i is very low, i.e., $Pr(d^i > \bar{d}) \leq \epsilon \ \forall i$

(for simulation exercises reported in this paper, ϵ has been taken to be 0.001). Note that Π is a stochastic and symmetric (i.e., also doubly stochastic) matrix due to the above construction procedure.

Definition 2.5: (Hotspot Model) A hotspot (i.e., a region where threats may exist) is modeled as a map for probability of detecting the threat.

Let the probability of detection of a hotspot be denoted by P_D, which attains the maximum at the center of the hotspot and decays to zero linearly with distance from the center in a radially symmetric manner. In the present context, a hotspot is detected only by agents proximal to them. A hotspot is characterized by the following two parameters:

- The maximum probability of detection of the threat, P_{Dmax} ($= 0.8$ in this study)
- The effective radius (r_{hs}) of the circular region within which $P_D > 0.5$, i.e., agents further than a distance of r_{hs} from the center of the hot-spot have less than 0.5 probability of detecting the threat.

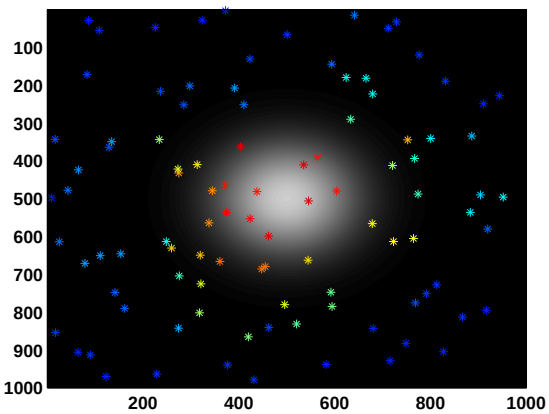


Fig. 1. Illustration of a hotspot with radially symmetric strength; probability of detection is maximum at the center and decays to zero linearly with distance from the center

The length scale $\lambda = r_{hs}/L$ of a hotspot is a non-dimensional quantity, where L is the side length of the operational area; the parameter $\lambda = 0.1$ in this paper. Detection depends on the proximity of the agent to the center of the hotspot, i.e., the value of P_D at its current location. An illustration of this scenario is presented in Fig. 1. Note that the hotspot model only considers the Type 2 (i.e., missed detection) error of a sensor; the effects of the Type 1 (i.e., false alarm) error are not considered here for the sake of model simplicity.

2) Decentralized Strategy: The decentralized strategy proposed here involves two characteristic variables associated with each agent. The first variable is called the *state characteristic function* that signifies whether an agent has detected a hotspot or not. The second variable is called the *agent measure function* that signifies the level of awareness or belief of an agent regarding the presence of a hotspot in the surveillance region. Formal definitions are presented below.

Definition 2.6: (State characteristic function) The state characteristic function (χ) of the agent population is defined as $\chi : Q \to \{0, 1\}$, where Q denotes the set of agents (nodes) and $\chi^i = 1$ signifies that the agent i has detected a hotspot itself and $\chi^i = 0$ denotes otherwise.

Definition 2.7: (Agent measure function) The agent measure function (ν) of the agent population is defined as a real measure $\nu : Q \to [0, 1]$, where Q denotes the set of agents (nodes). ν^i encodes the level of awareness or belief that agent i has about the existence of a hotspot in the operational area. $\nu^i = 0$ signifies that agent i has no knowledge regarding a hotspot in the area, whereas $\nu^i = 1$ means that agent i has maximum belief that a hotspot exists in the area of surveillance.

Based on the current state characteristics functions (χ) and measure functions (ν) of the agent population, synchronous updating of measures are updated for all agents after the expiry of one message lifetime L_m. Naturally, L_m is homogeneous in the agent population. Although the global objectives can be achieved through asynchronous updating with heterogeneous distribution of L_m, a simpler condition is considered here for the sake of analytical tractability, as explained below.

If an agent i detects a hotspot, then the state characteristic function is maintained at $\chi^i = 1$ till the next global measure updating occurs even if the agent does not see the hotspot anymore for the remaining part of the same message lifetime. It is noted that, based on the discussion up to this point, Π, ν and χ are functions of the slow time-scale τ as discussed earlier in Section 2-A.

In the above setting, a decentralized strategy for measure updating in the mobile-agent population is introduced below in terms of a user-defined control parameter $\theta \in (0, 1]$.

$$\nu^i_\theta|_{\tau+1} = (1 - \theta) \sum_{j \in \{i\} \cup Nb(i)} \Pi_{ij}|_\tau \nu^j_\theta|_\tau + \theta \chi^i|_\tau \tag{1}$$

where $Nb(i)$ denotes the set of agents in the neighborhood of agent i i.e., agents that communicate with the agent i during the time span τ and $\tau + 1$. It is noted that that while computing the future (awareness or belief) measure of an agent, the parameter θ controls the trade-off between the effects of current self-observation and current measures of all agents.

Expansion of Eqn. (1) yields:

$$\nu_\theta^i|_{\tau+1} = (1-\theta)\left[(1-\beta d^i)\nu_\theta^i|_\tau + \sum_{j\in Nb(i)} \beta\nu_\theta^j|_\tau\right] + \theta\chi^i|_\tau \tag{2}$$

The above equation (2) signifies that the self-influence for an agent reduces with increase of its degree. In other words, the more neighbors an agent communicates to, the less it relies just on its own observation. The evolution of a measure for an agent over slow-time epochs is illustrated in Fig. 2. In the vector notation, the dynamics can be expressed as:

$$\nu_\theta|_{\tau+1} = (1-\theta)\Pi|_\tau \nu_\theta|_\tau + \theta\chi|_\tau \tag{3}$$

The recursive relation in the Eqn. (3) above is expanded as:

$$\begin{aligned}\nu_\theta|_{\tau+1} = (1-\theta)^{\tau+1}[\Pi|_\tau \Pi|_{\tau-1}\cdots\Pi|_0]\nu_\theta|_0 + \theta\chi|_\tau \\ +\theta(1-\theta)\Pi|_\tau\chi|_{\tau-1} + \theta(1-\theta)^2\Pi|_\tau\Pi|_{\tau-1}\chi|_{\tau-2} \\ +\cdots+\theta(1-\theta)^\tau\Pi|_\tau\Pi|_{\tau-1}\cdots\Pi|_1\chi|_0\end{aligned} \tag{4}$$

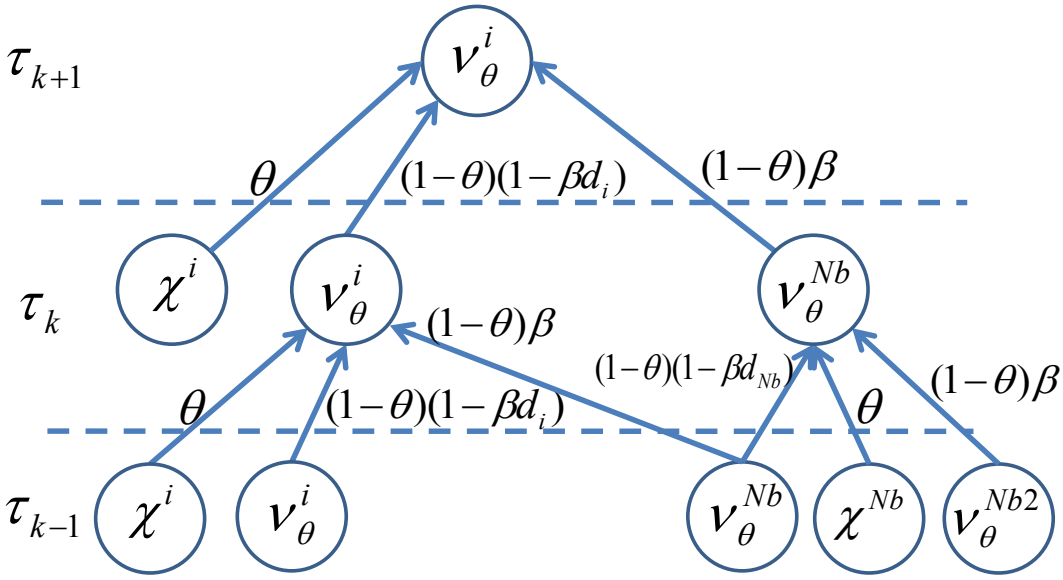


Fig. 2. Illustration of Generalized Gossip strategy: evolution of measure ν_θ of agent i over slow-time epochs; superscript Nb denotes immediate neighbors and $Nb2$ denotes second-order neighbors

Thus, this policy is simply a gossip algorithm with varying input $\chi|_\tau$ and varying network topology represented by $\Pi|_\tau$. The memory of a past input fades as a function of the parameter θ. Due to this notion, the above policy can be called a *generalized gossip algorithm* with θ as the generalizing parameter. The decentralized strategy is described in an algorithmic form below:

Algorithm 2: Generalized Gossip policy

Choose global parameters θ, β
τ_{end}: Total simulation time in the slow scale
$\tau = 1$
$\nu_\theta|_\tau = 0$ Initialize Measure values for all agents
$\chi|_\tau = 0$ Initialize State characteristics function for all agents
while $\tau < \tau_{end}$ **do**
  Evaluate $\chi|_\tau$ based on observations made by agents during slow-time epoch τ
  for all Agent i **do**
    Determine degree d^i for current slow-time epoch

Current observation: $\chi^i|_\tau$
Current measure value: $\nu^i_\theta|_\tau$
Collect current measure values from neighbors: $\nu^j_\theta|_\tau \ \forall j \in Nb(i)$
Compute future measure value:
$\nu^i_\theta|_{\tau+1} = (1-\theta)\left[(1-\beta d^i)\nu^i_\theta|_\tau + \sum_{j\in Nb(i)} \beta\nu^j_\theta|_\tau\right] + \theta\chi^i|_\tau$
end for
$\chi|_{\tau+1} = 0$ Reset State characteristics function for all agents
$\tau \leftarrow \tau + 1$
end while

3. Convergence of Statistical Moments

The convergence results presented here naturally involve expected quantities due to the inherent stochastic nature of the problem. Thus, even in the steady state, ν_θ will always fluctuate in the slow time-scale due to the fluctuations in Π and χ. However, interesting observations regarding slow time-scale evolution of the system can be made in terms of statistical moments of ν_θ computed over the agent population. In this paper, both average (over agents) $\mathbb{M}_a[\cdot]$ and variance (over agents) $\mathbb{V}_a[\cdot]$ of ν_θ are considered at a steady state. Note, $\nu_\theta|_\tau$ at a slow time instant τ is an N-dimensional vector, where N is the number of agents in the population. Hence, $\mathbb{M}_a[\nu_\theta|_\tau]$ and $\mathbb{V}_a[\nu_\theta|_\tau]$ are respectively scalar average and variance values, where $\nu_\theta|_\tau$ is considered as a random variable with N samples. In general, the functions $\mathbb{M}_a[\cdot]$ and $\mathbb{V}_a[\cdot]$ are defined on an N dimensional column vector $\mathbf{x} = [x_1, x_2, ..., x_N]^T$ as follows:

$$\mathbb{M}_a(\mathbf{x}) = \frac{1}{n}\mathbf{1}\mathbf{x} = \mathbf{x}^{avg} \tag{5}$$

where $\mathbf{1}$ is a row vector with all elements as 1. After the mean is subtracted, let the resulting vector be denoted as $\tilde{\mathbf{x}}$, i.e., $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}^{avg}\mathbf{1}^T$. Therefore, $\mathbb{V}_a(\mathbf{x}) = \tilde{\mathbf{x}}^T\tilde{\mathbf{x}}$.

A. Convergence of Measure Average over Agents

Recall the system dynamics as given in Eqn. (3).

$$\nu_\theta|_{\tau+1} = (1-\theta)\Pi|_\tau\nu_\theta|_\tau + \theta\chi|_\tau \tag{6}$$

The following equation is obtained by pre-multiplying $\frac{1}{n}\mathbf{1}$ on both sides of Eqn. (6).,

$$\nu^{avg}_\theta|_{\tau+1} = (1-\theta)\nu^{avg}_\theta|_\tau + \theta\chi^{avg}|_\tau \tag{7}$$

Note, $\mathbf{1}\Pi|_\tau = \mathbf{1}$, as $\Pi|_\tau$ is doubly stochastic. Expanding Eqn. (7), one obtains

$$\begin{aligned}\nu^{avg}_\theta|_{\tau+1} = (1-\theta)^{\tau+1}\nu^{avg}_\theta|_0 + \theta\chi^{avg}|_\tau \\ +\theta(1-\theta)\chi^{avg}|_{\tau-1} + \theta(1-\theta)^2\chi^{avg}|_{\tau-2} \\ + \cdots + \theta(1-\theta)^\tau\chi^{avg}|_0\end{aligned} \tag{8}$$

Considering the unrestricted 2-D random motion of the agents in the entire region, the ensemble expectation of $\chi^{avg}|_k$ is denoted as $E[\chi^{avg}]\ \forall k$ (i.e., no time dependency). In this case, $E[\chi^{avg}]$ signifies the fraction of agents that visit the hotspot on the average. Therefore, it is evident that, with a constant strength of the hotspot, $E[\chi^{avg}]$ remains constant over time. Taking (ensemble) expectation on both sides of Eqn. 8, the following relation is obtained at a steady state (as $\tau \to \infty$).

$$\begin{aligned}E[\nu^{avg}_\theta|_\infty] &= \theta[1 + (1-\theta) + (1-\theta)^2 + \cdots]E[\chi^{avg}] \\ &= \theta[1-(1-\theta)]^{-1}E[\chi^{avg}] \\ &= E[\chi^{avg}] \ \text{ for } \theta \in (0,1]\end{aligned} \tag{9}$$

Therefore, using the notation of steady-state average (over agents) introduced before, the steady-state expected measure average (over agents) is obtained as:

$$E[\mathbb{M}_a(\nu_\theta)] = E[\mathbb{M}_a(\chi)] \tag{10}$$

Convergence of the average measure to average χ implies that, at a steady state, the sum of χ values over agents is same as the sum of ν values over agents. In general, the physical significance is that the detection decision of a hotspot by few agents is being redistributed as awareness over a (possibly) larger number of agents, where the total awareness measure is conserved. From this perspective, it is interesting to know the nature of measure distribution in the agent population and measure variance

(over agents) provides an insight in this aspect. For example, an extreme case would be when measure variance is zero, that is all agents have the same measure and it is equal to the average measure of the population. In literature, this scenario is known as *consensus*. An opposite extreme case is when there is no awareness propagation; only those agents that have detected a hotspot (i.e., have nonzero χ) have nonzero measure. The measure variance is equal to the variance of χ in this case and the hotspot can be localized very well following the measure distribution due to a sharp localization gradient. Thus, measure distribution essentially dictates a tradeoff between *Propagation Radius* and *Localization Gradient* and variance of ν over agents quantifies the position of the system in this tradeoff scale.

B. Convergence of Measure Variance over Agents

For variance calculation, consider post-multiplication of $\mathbf{1}^T$ on both sides of Eqn. (7),

$$\begin{aligned} \nu_\theta^{avg}|_{\tau+1}\mathbf{1}^T &= (1-\theta)\nu_\theta^{avg}|_\tau \mathbf{1}^T + \theta\chi^{avg}|_\tau \mathbf{1}^T \\ \Rightarrow \nu_\theta^{avg}|_{\tau+1}\mathbf{1}^T &= (1-\theta)\nu_\theta^{avg}|_\tau \Pi|_\tau \mathbf{1}^T + \theta\chi^{avg}|_\tau \mathbf{1}^T \end{aligned} \tag{11}$$

The above equation presents the mean dynamics for the system. Now, the following equation is obtained by subtracting the mean dynamics in Eqn. (11) from the system equation in Eqn. (6).

$$\tilde{\nu}_\theta|_{\tau+1} = (1-\theta)\Pi|_\tau \tilde{\nu}_\theta|_\tau + \theta\tilde{\chi}|_\tau \tag{12}$$

For calculation of variance (over agents),

$$\begin{aligned} (\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1}) &= (1-\theta)^2(\tilde{\nu}_\theta|_\tau)^T(\Pi|_\tau)^T(\Pi|_\tau)(\tilde{\nu}_\theta|_\tau) \\ &+\theta^2(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau) + 2\theta(1-\theta)(\tilde{\nu}_\theta|_\tau)^T(\Pi|_\tau)^T(\tilde{\chi}|_\tau) \end{aligned} \tag{13}$$

At this point, one needs to take ensemble expectation on both sides. Since closed form results may not be analytically tractable in general, certain assumptions are made that may restrict the problem scenario to some extent. It is evident from the discussion till now that there exists two fundamental aspects of the problem, one related to network evolution and the other related to agent state dynamics and they can have very different time scales. Let us consider a case, where the time scales of these two aspects are comparable, which means that, at each slow-time epoch τ (when the agent measures are updated), the system has an independent agent interaction matrix Π as well as an independent state characteristic vector χ. Physically, this requires the agents to move fast enough or the message lifetime to be large enough so that temporal correlations die out between two slow-time epochs. This case is referred to as the Congruous Time Scale (CTS) case in this paper. Formally, the following assumptions are made for the CTS case.

- By problem setup, Π at any slow-time epoch depends on the mobility characteristics of the agent population and the message lifetime L_m, neither of which is affected by the presence of a hotspot. On the other hand, the vector χ at any slow-time epoch captures the information regarding hotspot detection by agents irrespective of inter-agent communication. Hence, it is assumed that $\Pi|_i$ and $\chi|_k$ are independent for every i and k.
- In this setup, the motion dynamics of an agent take place at a fast time-scale, denoted by t, and Π captures the inter-agent communication characteristics (due to agent motion) for a window of fast time-scale. Now, for a large enough window (i.e., a large value of L_m), it is assumed that the fast time-scale mobility correlation dies out within a relatively short period. As a consequence, $\Pi|_i$ and $\Pi|_j$ become mutually independent for every i and j.
- The agents move fast enough (or in other words, the hotspot length scale is reasonably small compared to the scale of agent motion) such that $\chi|_i$ and $\chi|_j$ are independent for every i and j.

The first two assumptions are feasible under fairly general conditions, whereas the third one requires a special condition of agent mobility. The simulation scenario presented in Section 4 provides an example. Future studies will explore the feasibility conditions of these assumptions in greater details.

By an application of Eqn. (13) under the above assumptions, it follows that the ensemble expectation (given $\tilde{\nu}_\theta|_\tau$) on both sides is:

$$\begin{aligned} E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})|\tilde{\nu}_\theta|_\tau] = \\ (1-\theta)^2(\tilde{\nu}_\theta|_\tau)^T E[(\Pi|_\tau)^T(\Pi|_\tau)](\tilde{\nu}_\theta|_\tau) + \\ \theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] + 2\theta(1-\theta)(\tilde{\nu}_\theta|_\tau)^T E[(\Pi|_\tau)^T]E[(\tilde{\chi}|_\tau)] \end{aligned} \tag{14}$$

Since all the agents perform a random walk motion, they are equally likely to visit the hot spot. This implies that $E[(\tilde{\chi}|_\tau)] = 0$. Furthermore,

$$(1-\theta)^2(\tilde{\nu}_\theta|_\tau)^T E[(\Pi|_\tau)^T(\Pi|_\tau)](\tilde{\nu}_\theta|_\tau) \geq 0 \tag{15}$$

Therefore, for the lower bound

$$\begin{aligned} E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})|\tilde{\nu}_\theta|_\tau] &\geq \theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] \\ \Rightarrow E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})] &\geq \theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] \end{aligned} \tag{16}$$

The expected (steady-state) variance is expressed as: $E\left[\mathbb{V}_a[\nu_\theta]\right] = E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})]$. Using a similar notation for χ, one has:

$$\frac{E\left[\mathbb{V}_a[\nu_\theta]\right]}{E\left[\mathbb{V}_a[\chi]\right]} \geq \theta^2 \tag{17}$$

Note, by construction $\tilde{\nu}_\theta|_\tau \perp \mathbf{1}^T$ [20]. Also, $\mathbf{1}$ is the stationary vector (left eigenvector corresponding to the unity eigenvalue) of a doubly stochastic matrix. Therefore,

$$(\tilde{\nu}_\theta|_\tau)^T E[(\Pi|_\tau)^T(\Pi|_\tau)](\tilde{\nu}_\theta|_\tau) \leq \Lambda_2(\tilde{\nu}_\theta|_\tau)^T(\tilde{\nu}_\theta|_\tau) \tag{18}$$

where, $\Lambda_2 = \lambda_2\left(E[(\Pi|_\tau)^T(\Pi|_\tau)]\right)$. Therefore, for the upper bound

$$\begin{aligned} E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})|\tilde{\nu}_\theta|_\tau] &\leq (1-\theta)^2\Lambda_2(\tilde{\nu}_\theta|_\tau)^T(\tilde{\nu}_\theta|_\tau) \\ &\quad +\theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] \\ \Rightarrow E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})] &\leq (1-\theta)^2\Lambda_2 E[(\tilde{\nu}_\theta|_\tau)^T(\tilde{\nu}_\theta|_\tau)] \\ &\quad +\theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] \end{aligned} \tag{19}$$

At a steady-state, $E\left[\mathbb{V}_a[\nu_\theta]\right] = E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})] = E[(\tilde{\nu}_\theta|_\tau)^T(\tilde{\nu}_\theta|_\tau)]$. Therefore,

$$\begin{aligned} E\left[\mathbb{V}_a[\nu_\theta]\right]\left[1-(1-\theta)^2\Lambda_2\right] &\leq \theta^2\mathbb{V}_a[\chi] \\ \Rightarrow \frac{E\left[\mathbb{V}_a[\nu_\theta]\right]}{E\left[\mathbb{V}_a[\chi]\right]} &\leq \frac{\theta^2}{1-(1-\theta)^2\Lambda_2} \end{aligned} \tag{20}$$

Note, $\theta \in (0,1]$ and $\Lambda_2 \in [0,1]$. Figure 3 presents the plot of upper bounds of the variance ratio $\frac{E[\mathbb{V}_a[\nu_\theta]]}{E[\mathbb{V}_a[\chi]]}$ with θ for three

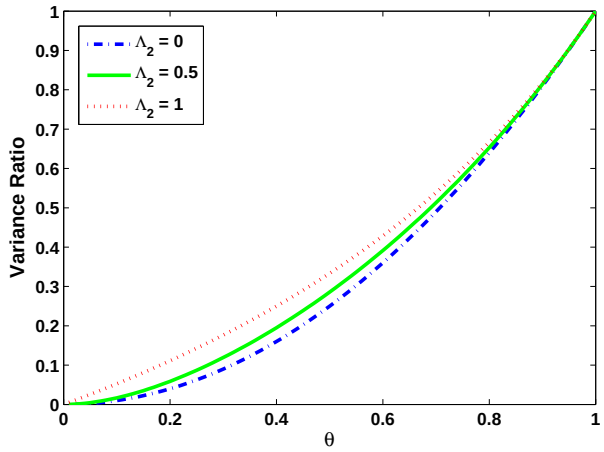


Fig. 3. Upper bounds of the variance ratio $\frac{E[\mathbb{V}_a[\nu_\theta]]}{E[\mathbb{V}_a[\chi]]}$ as a function of θ and $\Pi|_\tau$ under CTS assumptions; lower bound is independent of Λ_2 and coincides with the upper bound for $\Lambda_2 = 0$

possible values of Λ_2. Note that the lower bound of the variance ratio is independent of Λ_2 and coincides with the upper bound for $\Lambda_2 = 0$.

It is understood that CTS is a special case in the spectrum of time-scale comparison of network evolution and the associated information propagation. In the CTS case, these time scales are congruous or comparable. On the other end of this spectrum, one can consider a situation where the two time scales are very different such that, the network evolution (the slow dynamics) and the agent state updating (the fast dynamics) can be treated independently as it is done in the *Singular Perturbation theory*. The problem becomes much simpler in this case as one may assume that Π and χ remain time-invariant over the course of transience in the agent state dynamics, i.e., agent measures converge before there is a change in Π and χ. This case is referred

to as the Disparate Time Scale (DTS) case in this paper. Under the DTS assumptions, Π and χ are not necessarily functions of τ. Therefore, from Eqn. (4), as $\tau \to \infty$, one has:

$$\begin{aligned}\nu_\theta|_\infty = \theta\chi + \theta(1-\theta)\Pi\chi + \theta(1-\theta)^2\Pi^2\chi \\ +\theta(1-\theta)^3\Pi^3\chi\cdots\end{aligned} \tag{21}$$

The following equation is obtained by subtracting the mean dynamics from Eqn. (21).

$$\begin{aligned}\tilde{\nu}_\theta|_\infty = \theta\tilde{\chi} + \theta(1-\theta)\Pi\tilde{\chi} + \theta(1-\theta)^2\Pi^2\tilde{\chi} \\ +\theta(1-\theta)^3\Pi^3\tilde{\chi}\cdots\end{aligned} \tag{22}$$

Using the above equation, the measure variance over agents is calculated as:

$$\begin{aligned}\mathbb{V}_a[\nu_\theta] = \theta^2\tilde{\chi}^T\tilde{\chi} + \theta^2(1-\theta)\tilde{\chi}^T\Pi\tilde{\chi} + \theta^2(1-\theta)\tilde{\chi}^T\Pi^T\tilde{\chi} \\ +\theta^2(1-\theta)^2\tilde{\chi}^T\Pi^T\Pi\tilde{\chi} + \theta^2(1-\theta)^2\tilde{\chi}^T\Pi^2\tilde{\chi} \\ +\theta^2(1-\theta)^2\tilde{\chi}^T(\Pi^2)^T\tilde{\chi}\cdots\end{aligned} \tag{23}$$

As Π is symmetric, one has:

$$\begin{aligned}\mathbb{V}_a[\nu_\theta] = \theta^2\tilde{\chi}^T\tilde{\chi} + 2\theta^2(1-\theta)\tilde{\chi}^T\Pi\tilde{\chi} \\ +3\theta^2(1-\theta)^2\tilde{\chi}^T\Pi^2\tilde{\chi}\cdots\end{aligned} \tag{24}$$

Since Π^k s are positive definite for $k \in \mathbb{N}$, the lower bound is obtained as:

$$\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]} \geq \theta^2 \tag{25}$$

Using the same logic as before, it is evident that $\tilde{\chi}^T\Pi^k\tilde{\chi} \leq \lambda_2(\Pi^k)\tilde{\chi}^T\tilde{\chi}$ for $k \in \mathbb{N}$. Also, $\lambda_2(\Pi^k) = \lambda_2^k(\Pi)$ and $\lambda_2(\Pi)$ is denoted simply as λ_2 in the sequel. Therefore,

$$\begin{aligned}\mathbb{V}_a[\nu_\theta] \leq \theta^2\mathbb{V}_a[\chi] + 2\theta^2(1-\theta)\lambda_2\mathbb{V}_a[\chi] \\ +3\theta^2(1-\theta)^2\lambda_2^2\mathbb{V}_a[\chi]\cdots\end{aligned} \tag{26}$$

Based on the infinite sum, the upper bound is obtained as:

$$\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]} \leq \frac{\theta^2}{[1-(1-\theta)\lambda_2]^2} \tag{27}$$

Note, $\theta \in (0, 1]$ and $\lambda_2 \in [0, 1]$. The upper bound for the variance ratio calculated above is valid for a particular Π. Figure 4

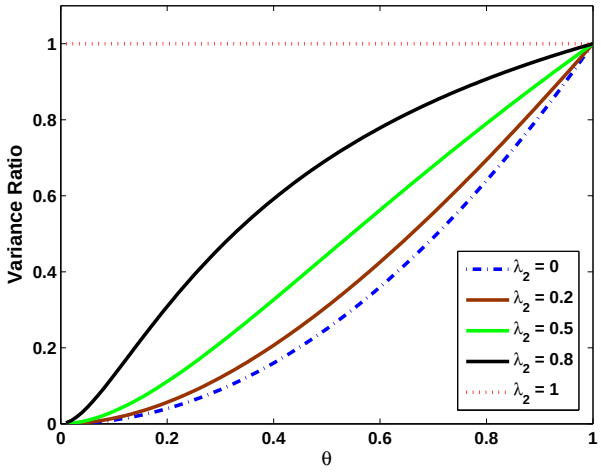
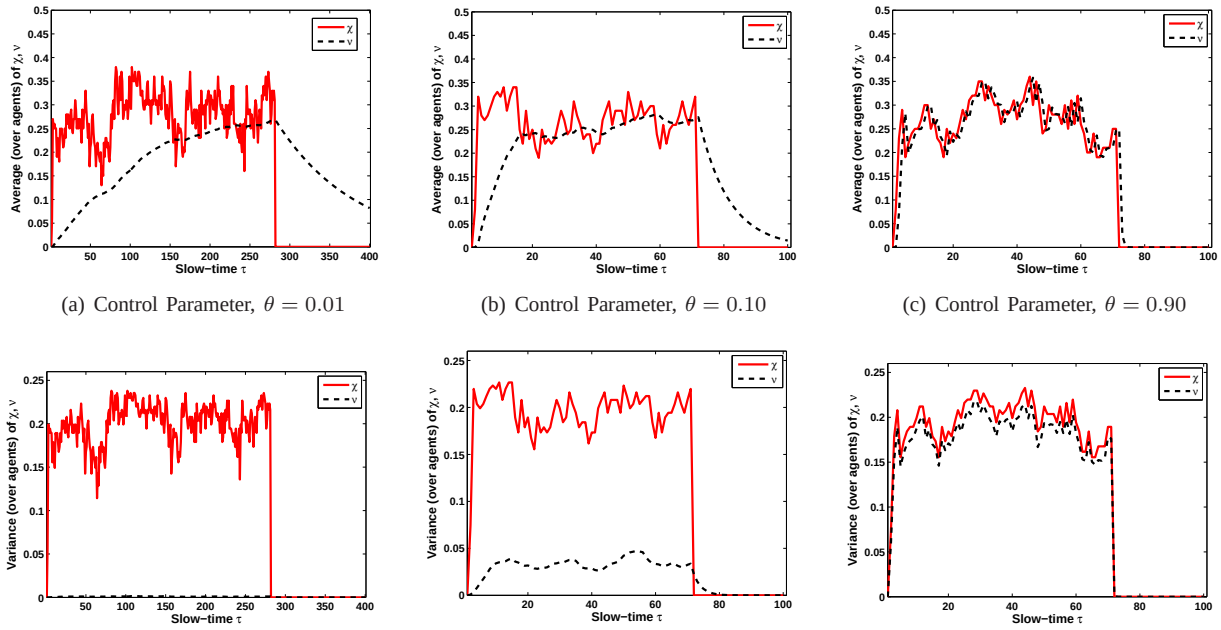


Fig. 4. Upper bounds of the variance ratio $\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]}$ as a function of θ and Π under DTS assumptions; lower bound is independent of λ_2 and coincides with the upper bound for $\lambda_2 = 0$

presents the plot of upper bounds of the variance ratio $\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]}$ with θ for three possible values of λ_2. Note that the lower bound of the variance ratio is independent of λ_2 and coincides with the upper bound for $\lambda_2 = 0$.



(a) Control Parameter, $\theta = 0.01$ (b) Control Parameter, $\theta = 0.10$ (c) Control Parameter, $\theta = 0.90$

(d) Control Parameter, $\theta = 0.01$ (e) Control Parameter, $\theta = 0.10$ (f) Control Parameter, $\theta = 0.90$

Fig. 5. Propagation of global awareness for hotspot length scale $\lambda = 0.10$ on a mobile-agent network with message lifetime $L_m = 30$. Plates (a), (b), (c) show the time evolution of average (over agents) of χ and ν and plates (d), (e), (f) show the time evolution of variance (over agents) of χ and ν; hotspot is switched on at $\tau = 2$ and switched off at $\tau = 280$ for $\theta = 0.01$ and at $\tau = 70$ for $\theta = 0.10, 0.90$.

It is observed in both cases that the upper bound and lower bound coincide as θ approaches extreme values, 0 or 1 and as seen in Section 4, $\mathbb{V}_a[\nu_\theta] \to 0$ as $\theta \to 0$ and $\mathbb{V}_a[\nu_\theta] \to \mathbb{V}_a[\chi]$ as $\theta \to 1$. In other words, the agent population approaches *consensus* as $\theta \to 0$ (but $\neq 0$). In this case, although the entire population becomes aware of the hotspot, there is no localization gradient as every agent has the same measure. On the other hand, with $\theta \to 1$, the localization gradient improves at the cost of propagation radius. In general, $\mathbb{V}_a[\nu_\theta]$ decreases with a reduction in θ. The other system component affecting the variance ratio is the Π matrix. In both CTS and DTS cases, this effect is realized through the second largest eigenvalue of Π. Reduction in the magnitude of the second largest eigenvalue of Π signifies more connectivity among agents. This fact explains the reduction in variance ratio with a decrease in the second largest eigenvalue.

4. Validation by Numerical Simulation

An example problem of surveillance and reconnaissance is presented in this section, which involves the mobile multi-agent network and the interaction policy as explained in Sections 2-A and 2-B.

A. Problem Statement

Let us consider a surveillance and reconnaissance mission for a region of area A performed by N mobile agents, where each agent has a radius of communication R. The agents are moving in the region with a 2-D random walk fashion with speed (i.e., displacement per unit time) v. The individual mission goal of the agents is to detect existence of any possible hotspot in the region and communicate this information to their neighboring agents. The information that an agent wants to communicate based on its recent observation, has a message lifetime L_m units in the fast time-scale (corresponding to the agent's motion). Thus, an epoch τ in the slow time-scale spans over L_m units of time in the fast time-scale. However, detection of a hotspot does not affect mobility characteristics of agents. The global mission objective is to control the extent of information propagation across the mobile agent network. For the simulation study, the parameters are chosen as follows: $A = 1000^2$, $N = 100$, $R = 100$, and $v = 20$. The message lifetime (L_m) is taken to be 30 time units. As described earlier, the state characteristic function χ^i of agent i becomes 1 upon detecting a hotspot; otherwise, χ^i remains 0. Also given the detection model described in Section 2-B, probability of successful detection depends on the proximity of the agent to the center of the hotspot. After the expiry of message lifetime L_m, the χ value of an agent resets to 0. The values of agent measure (ν) are updated based on the agent interaction policy described earlier.

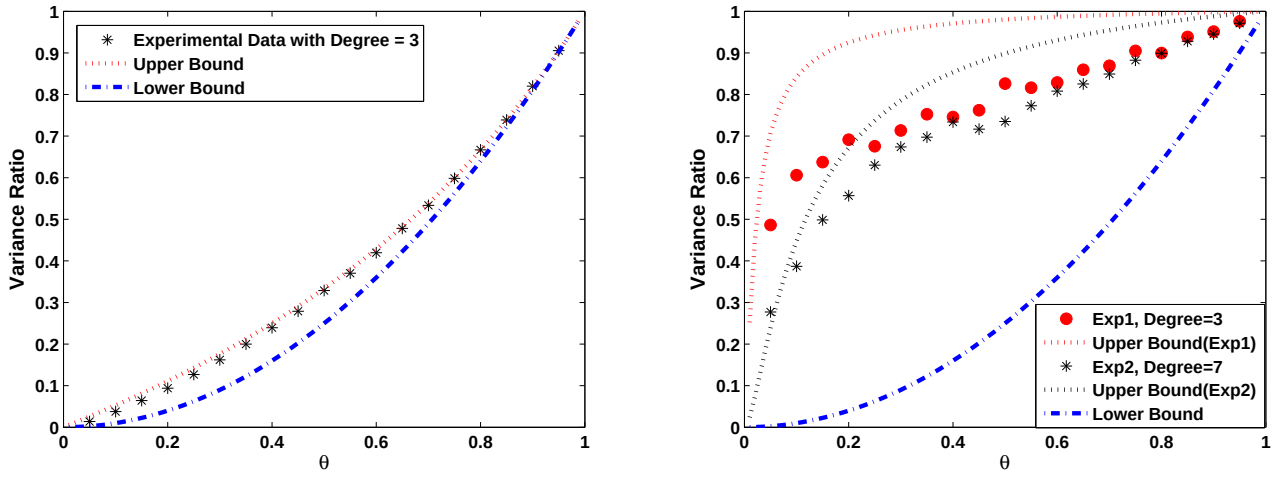
Simulation runs have been conducted with different values of θ. In this paper, observations regarding convergence of statistical moments (e.g., mean and variance) of ν_θ with $\theta = 0.01, 0.10$, and 0.90 are presented. Top three plates, (a), (b) and (c), in Fig. 5 present the time evolution of average (over agents) and the bottom three plates, (d), (e), (f), present the corresponding variance (over agents) for time-series of χ and ν. The hotspot is switched on at $\tau = 2$ for all experiments and switched off at $\tau = 280$ for $\theta = 0.01$ and at $\tau = 70$ for $\theta = 0.10$ and 0.90.

B. Results & Discussions

It is observed in plates (a), (b) and (c) of Fig. 5 that, after the appearance of hotspot, the average (over agents) ν converges to the average (over agents) χ at the steady state for three different values of the control parameter θ, where the convergence time decreases with an increase in θ. The above observation is explained below.

It follows from Eqn. (3) that the system dynamics depend on the largest eigenvalue of $(1-\theta)\Pi|_\tau$. Since $\Pi|_\tau$ is an irreducible stochastic matrix, Perron-Frobenius theorem ensures that its largest eigenvalue is 1; thus, the largest eigenvalue of $(1-\theta)\Pi|_\tau$ is $(1-\theta)$. Therefore, it is expected that the convergence time will increase with decrease in θ. Moreover, the first order dynamics can be observed in the time evolution of average ν; this can be attributed to the uniqueness of the largest eigenvalue of Π. Plates (d), (e), (f) of Fig. 5 show that the steady state variance (over agents) of ν increases with increase in θ; also, $\mathbb{V}_a[\nu] \to 0$ as $\theta \to 0$ and $\mathbb{V}_a[\nu] \to \mathbb{V}_a[\chi]$ as $\theta \to 1$. These observations regarding the dependence of steady-state statistical moments of the agent measure on system parameters further validate the analytical claims made in the previous section.

Figure 6 shows the results of numerical simulation for verification of upper and lower bounds on the variance ratios $\frac{E[\mathbb{V}_a[\nu_\theta]]}{E[\mathbb{V}_a[\chi]]}$ and $\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]}$ for CTS and DTS assumptions, respectively. The results for CTS are presented in Fig. 6(a), where the simulation results to closely follow the upper bound for this particular case. While the expected degree of the network is kept as 3, high speed ($v \sim 100$) is assumed for agents to achieve the conditions described in the CTS assumptions. Results of numerical simulation for DTS are presented in Fig. 6(b) that shows the data for two cases with expected degree of the network as 3 and 7. The agent speed is kept nonzero but very low ($v \sim 5$) to achieve the conditions described under the DTS assumptions.



(a) Variance ratio $\frac{E[\mathbb{V}_a[\nu_\theta]]}{E[\mathbb{V}_a[\chi]]}$ under CTS assumptions

(b) Variance ratio $\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]}$ under DTS assumptions

Fig. 6. Simulation-based verification of bounds on variance under CTS and DTS assumptions

Remark 4.1: It is noted that the upper bounds on the variance ratio for both CTS and DTS cases are found to be functions of the second largest eigenvalue of the agent interaction matrix Π which, in turn, is a function of the degree of the network. However, the degree is not the only network parameter that determines the decision propagation characteristics as it is observed in the simulation for both CTS and DTS cases. Therefore, it is evident that, apart from the network degree, comparability between time scales of network evolution and agent state dynamics also plays a key role in determining the network system characteristics.

5. Summary, Conclusions and Future Work

This paper addresses the problem of decision/awareness propagation in a mobile-agent network environment for surveillance and reconnaissance. A distributed decision propagation algorithm has been constructed based on the concepts of recently

developed language-measure theory [14][15]. A completely decentralized implementation of this algorithm is shown to be useful for propagation of awareness regarding a local hotspot in the operational area. Analytical results have been obtained for convergence of (awareness level) measure distribution in the agent population. A (user-defined) critical parameter θ controls the tradeoff between the propagation radius and the localization gradient, where θ has both temporal effects (e.g., convergence time) and ensemble effects (e.g., the measure distribution characteristics in the agent population). In this setting, *consensus* can be achieved as $\theta \to 0$.

Two cases, congruous time scale (CTS) and disparate time scale (DTS), relating the time scales of network topology and agent interaction are presented and validated by numerical simulation on a test bed for a typical example problem. In this algorithm, the system is reset automatically upon removal of a hotspot. Another advantage of this approach is that it naturally extends to multiple hotspot scenarios. However, it will be interesting to investigate such conditions with both homogeneous and heterogeneous hotspots. Following are the future research directions that are currently being pursued:

- Analytical evaluation of the expected characteristics of Π (e.g., the second largest eigenvalues), given the expected characteristics of the proximity network;
- Analysis of convergence dynamics/time under the current framework;
- Investigation of scenarios with asynchronous measure updating and heterogeneous message lifetime distribution;
- Exploration of the feasibility conditions of the assumptions made for the CTS case and the possibility of relaxing these assumptions for variance calculation;
- Identification of quantitative rules to distinguish between a CTS case and DTS case;
- Evaluation of *Generalized Gossip* policy with more realistic hotspot detection model (e.g., inclusion of false alarm possibility);
- Extension of the decentralized policy presented here to be used for *event-triggered* or *self-triggered* co-operative control problems.

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APPENDIX A
ANALYSIS OF PROXIMITY NETWORKS

This appendix analyzes the effects of L_m on the time averaged proximity network topology that is primarily represented by the statistics of the degree of a network node. Although these definitions are straightforward for static networks, they need to be carefully constructed in the present context of dynamic networks.

Definition 5.1: (Degree and Degree Distribution) The degree k of a node (agent) is defined to be the number of network nodes to which it is connected; and the degree distribution $P(k)$ for a network is defined to be the probability distribution of degrees over the entire network. Let $P(k|L_m, i)$ be the distribution (computed over time) of the number of distinct nodes that communicate with a given node i within its message lifetime $L_m(i)$. The degree distribution is defined as:

$$P(k|\bar{L}_m) \triangleq \frac{1}{n(\bar{L}_m)} \sum_{i:L_m(i)=\bar{L}_m} P(k|\bar{L}_m, i) \tag{28}$$

where $n(\bar{L}_m)$ is number of nodes in the network with message lifetime $\bar{L}_m$. Finally, the overall network degree distribution is defined as the expected value of $P(k|L_m)$, i.e.,

$$P(k) \triangleq \frac{1}{N} \sum_{L_m} n(L_m)\ P(k|L_m) \tag{29}$$

where N is total number of nodes in the network.

It is reported in literature [7][21] that the degree distributions for proximity networks have different structures (e.g., Poisson distribution and power law distribution as in scale-free networks), depending on the parameters of the mobile-agent dynamics. For the parameters selected in Section 4, the time-averaged degree distribution $P(k|L_m)$ follows the Poisson distribution as explained below.

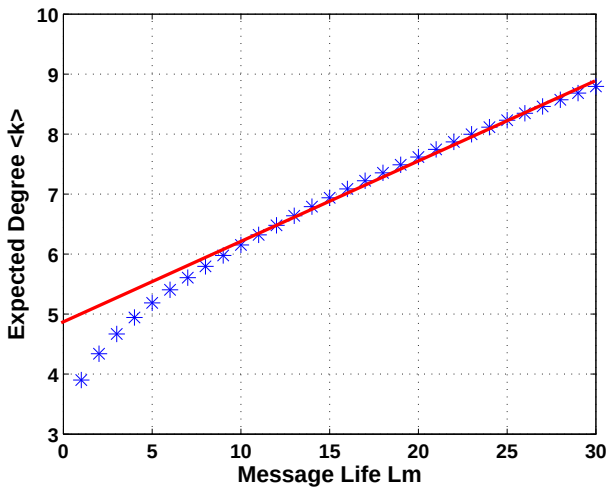

Fig. 7. Variation of expected degree $< k >$ of the network with homogeneous message life L_m.

Let the node i have the message lifetime L_m and let the probability that an agent j $(j \neq i)$ is within the zone of communication of i within the time window L_m be denoted as $p_{ij}(L_m)$ (that is obviously an increasing function of L_m). At a given stage of network evolution, let the probability that i and j communicate in the next time step be modeled as $\alpha g_i g_j (\leq 1)$, where g_l is called the *gregariousness* (i.e., the tendency to communicate with other agents) of agent l in accordance with social network literature [7][21] and let α be a parameter that incorporates spatial information of the network (e.g., agent density). Note that g_l is a function of the radius of communication and velocity of agent l.

With this model and assuming independent activity at each time step, the probability that the nodes i and j $(i \neq j)$ do not communicate within L_m is $(1 - \alpha g_i g_j)^{L_m}$. Therefore, $p_{ij}(L_m) = 1 - (1 - \alpha g_i g_j)^{L_m}$. The expected value $< k >_i$ of the

degree of the node i is obtained as:

$$< k >_i = \sum_{j=1}^{N} p_{ij}(L_m)$$
$$\simeq \alpha L_m g_i(\sum_j g_j) \quad for\ \alpha L_m \ll 1 \tag{30}$$

The assumption of $\alpha L_m \ll 1$ is realized if α is very small and at the same time L_m is not very large; a small α provides an upper bound on the maximum number of nodes that the node i can communicate in one time step; this is called the *exclusion constraint* [7]). In this paper, the radius of communication and velocity are kept invariant for every node, implying that all nodes share a uniform gregariousness g. Therefore, $p_{ij}(L_m)$ is independent of agent specifications i and j and is denoted as $p(L_m)$ or simply p. Also, all agents are assumed to have same message lifetime L_m. With these assumptions, numerical experiments are performed to calculate the expected degree $< k >$ of the network for various values of homogeneous L_m. Figure 7 shows the result obtained from these experiments and an approximately linear relation between $< k >$ and L_m (as derived before) is observed beyond $L_m = 9$. Now, with homogeneous p and L_m across the network, the degree distribution $P(k|L_m)$ is written as:

$$P(k|L_m) = \binom{N}{k} p^k (1-p)^{N-k}$$
$$\simeq \frac{< k >^k}{k!} e^{-<k>} \quad for\ N >> 1 \tag{31}$$

Figure 8 shows the degree distribution $P(k|L_m)$ obtained from numerical experiments performed for $L_m = 1, 20, 30$ to be Poisson in nature. Note that the degree distribution for $L_m = 1$ represents the characteristics of a static proximity network. However, it is shown in [7] that by choosing non homogeneous $p_{ij}(L_m)$, one may obtain other types of degree distributions

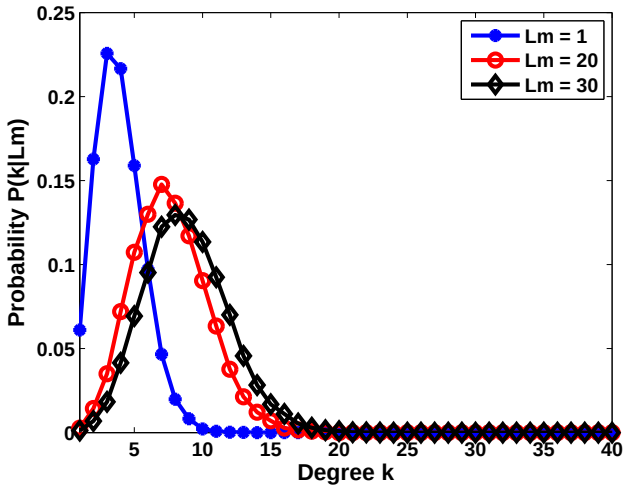

Fig. 8. Plots of degree distribution $P(k|L_m)$ of the mobile-agent network for $L_m = 1, 20, 30$.

(e.g., power-law distribution) as well. Thus, degree distribution and expected degree of the network (i.e., the expected network topology) can be controlled by varying L_m.

APPENDIX B
BASIC NOTIONS OF LANGUAGE-MEASURE THEORY

This appendix summarizes the concept of real measure of probabilistic regular languages generated by probabilistic finite state automata (PFSA) [14], [15]. However, since the topic of regular languages is beyond the scope of this paper, the concept of real measures have been restricted only to irreducible Markov chains.

A. Brief Review

Let a stationary Markov chain be denoted by the 3-tuple (Q, Π, χ), where $Q = \{q_1, q_2, ..., q_N\}$ is the set of states; the state transition function $\Pi : Q \times Q \rightarrow [0, 1]$ depicts the transition probabilities of the Markov chain and Π is expressed in the from of an $N \times N$ stochastic matrix; and the state characteristic function $\chi : Q \rightarrow \mathbb{R}$ assigns a signed real weight to each state. As the number of states is finite, the vector form of the characteristic function is written as $\chi = \left[\chi^1, \chi^2, ...\chi^N\right]^T$.

A real measure ν_θ^i for state i is defined as

$$\nu_\theta^i \triangleq \sum_{\tau=0}^{\infty} \theta \left(1-\theta\right)^\tau \mathbf{v}^i \Pi^\tau \chi \tag{32}$$

where $\theta \in (0, 1]$ is a user-specified parameter; and $\mathbf{v}^i$, defined as a $1 \times N$ vector $\left[v_1^i, v_2^i, ..., v_N^i\right]$, is given by

$$v_j^i = \begin{cases} 1 & if \quad i = j \\ 0 & if \quad i \neq j \end{cases} \tag{33}$$

Remark 5.1: (*Physical Significance of Real Measure*) Let us assume that the current state of the Markov precess is i, i.e., the $(1 \times N)$ state probability vector is $\mathbf{v}^i$. At an instant τ time-steps in the future, the state probability vector is given by $\mathbf{v}^i\Pi^\tau$. Further, the expected value of the characteristic function is given by $\mathbf{v}^i\Pi^\tau\chi$. The measure of state i, as described by Eqn. (32), is the weighted expected value of χ over the all the time-steps in future for a the Markov process that begins in state i. The weights for each time-step $(\theta\left(1-\theta\right)^\tau)$ is a function of the single parameter θ. In addition, the weights form a decreasing geometric series whose sum equals to 1. As a result, the measure ν_θ^i is a convex combination of all the elements of the χ vector and $\min_j \chi^j \leq \nu_\theta^i \leq \max_j \chi^j \ \forall i \leq N$.

The expression for the the measure in Eqn. (32) is expressed in an alternative equivalent form

$$\nu_\theta^i = \theta \mathbf{v}^i \left(I - (1-\theta)\Pi\right)^{-1} \chi \tag{34}$$

The inverse is guaranteed to exist for $\theta \in (0, 1]$.

From Eqn. (32), the measure of all the states denoted by the vector $\nu_\theta = \left[\nu_\theta^1, \nu_\theta^2, ..., \nu_\theta^N\right]^T$ is written as

$$\nu_\theta = \theta \left(I - (1-\theta)\Pi\right)^{-1} \chi \tag{35}$$

Remark 5.2: (*The effects of the parameter θ*) The parameter θ determines the weights $(\theta\left(1-\theta\right)^\tau)$ assigned to the expected characteristic function for time step τ. In particular, θ controls the the rate at which the weights decrease with increasing values of τ. Large values of θ forces the weights to decay rapidly, thereby placing more importance to the characteristic functions of states that are adjacent (connected with fewer hops) to the initial state i. In fact, $\theta = 1$ implies that $\nu_\theta^i = \chi^i$. On the other hand, small values of θ captures the interaction with a large neighborhood of connected states. As $\theta \rightarrow 0$, the dependence of on the initial state i slowly decays (provided Π is irreducible) and all the states converge to the same value of measure.