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Distributed Decision Propagation in Mobile Agent Networks★

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***Abstract*— This paper develops a distributed algorithm of decision/awareness propagation in mobile agent systems with a time varying network topology and threshold based agent interaction policy. While message broadcast duration or state updating interval is found to be an actuation parameter for changing time-averaged network topology, the threshold parameter in binary decision policy can be used to trigger or restrain the decision propagation. The influence of (large) seed size on the propagation phenomenon has been exploited to control the threat level threshold, beyond which the awareness propagates throughout the network.**

1. Introduction

Recently, analysis and development of distributed decision and control mechanisms in mobile agent networks have drawn much attention due to their immense relevance in engineering problems, such as surveillance and reconnaissance by autonomous vehicles with limited capabilities, trust establishments in mobile ad hoc networks (MANETs) [1] and threat monitoring by mobile sensor networks. In a resource constrained environment, mobile agents have potential advantages over static networks in terms of coverage and time-criticality. Irrespective of the specific characteristics, multi-agent problems are often categorized in the following major classes: flocking, consensus, formation, agreement and diffusion/propagation. This paper deals with global propagation of a localized awareness in a leaderless environment when the threat level crosses a predefined threshold in a robust and completely distributed manner.

In general, there are two aspects of interacting agent systems, namely (i) network topology and (ii) agent interaction dynamics. Network topology is inherently time varying in the present context, which makes the analysis of such complex systems much harder compared to their static counterparts. Usually, similar time-varying situations arise in social network studies [2] and they are modeled by various graphical structures, such as: multiple instances of uniform random graphs, scale-free networks and small world networks [3]. Synchronization problems have been solved for time-varying networks where essentially the network topology is modeled as fast switching among a finite number of instances of random graphs with same specifications [4]. However, all such models do not necessarily consider the agent mobility statistics or inter-agent communications due to proximity. Recently, the analysis of so called proximity networks [5] has been performed (also called moving neighborhood networks [6]) to model contact/collision based disease spreading. This may be considered as the first step towards analyzing the mobile agent scenario in an actual sense. This paper adopts such developments reported in social network studies, to analyze the expected network topology in a mobile agent framework.

Distributed agent interaction dynamics for decision propagation has several mechanisms available in literature, examples are game theoretic [1], biology inspired, physics inspired (Ising/Potts models) [7], bootstrap percolation [8] and majority voting [9]. However, a vast majority of these studies are restricted to the time-invariant network topologies. Irrespective of the mechanism used, phase transition phenomenon [10] is often observed in typical multi-agent problems mentioned before [11]. Watts [12], developed a simple threshold-based neighborhood interaction model, known as the binary decisions with externalities, that can be used to explain global cascading phenomena in large electrical power grids and opinion spreading in a large community. This model shows that global cascading from a very small localized seed in a complex (time-invariant) network can be triggered by a very small change in a single agent interaction parameter, which is known as a phase transition phenomenon. This paper adopts this mechanism in a time-varying network topology for global propagation of decision/awareness in mobile agent networks.

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The paper is organized in five sections including the present one. The representation of a mobile agent scenario in terms of proximity networks is formulated in Section 2. Section 3 explains the agent interaction policy and its necessary properties. Section 4 describes an example of engineering application regarding propagation of global awareness using the architecture developed in Section 2 and 3. Finally, the paper is summarized and concluded in Section 5 with recommendations of future work.

2. Proximity Networks: Representation of Mobile Agent Scenario

In the context of surveillance and reconnaissance by mobile agents, decentralized operations involving mobile agents is very useful. However, the network-theoretic modeling becomes challenging for such cases due to the time varying topology, which is a consequence of agent (node) mobility. Recent literature in social network studies have addressed similar problems by introducing the concept of proximity networks [5], [6]. This section describes a mobile agent scenario in terms of proximity networks and analyzes the relevant network parameters including degree distribution and coordination number (expected degree) [13].

A. Model Description

Let the area of a two dimensional (Euclidean) operational region be A. In the present case, A is assumed to be a square area with side length L, i.e., $A = L^2$. Initially, N agents are distributed randomly in the given area, and the agent density is defined as $\rho = N/A$. The uniform radius of communication for each agent is denoted by R, i.e., two agents can only communicate (to say, exchange their state values) when the distance between them is less than R. The agents move in a 2-D random walk fashion where the velocity v is same for all the agents in the current setup. The random walk is realized by independently choosing a random direction of motion by all agents at every time step. During its motion, every agent is assumed to broadcast a message (e.g., its own state) over a certain time window that is known as the message lifetime L_m. At the same time, the agent receives messages (e.g., state values) from other agents, which may come within the distance R. After expiry of a message, an agent possibly undergoes a state updating and then it starts broadcasting the new state for another window of message lifetime. Based on realistic scenarios, the following assumptions are made regarding the network parameters mentioned above. Firstly, the goal of an intelligent surveillance and reconnaissance by mobile agents mission is to monitor a sufficiently large area with relatively fewer agents, where the agents may not form a fully connected network in terms of communication. This fact translates into the assumption of $R/L << 1$, with sufficiently small ρ. Secondly, the consideration of physical motion of an agent is always a very expensive option. Hence, the length scale of the physical movement is kept small compared to the communication length scale, i.e., $x/R < 1$, where x denotes displacement in one time unit. These considerations lead to the assumption of $x/L << 1$. In the present study, the chosen parameters are: $L = 1000$, $N = 100$, $R = 100$, $x = 20$. If the message lifetime L_m is very small, then the effective network will be equivalent to multiple instances of static random graphs. On the other hand, the network eventually becomes fully connected as $L_m \to \infty$. Thus, to model realistic situations, L_m should be chosen appropriately based on the other network parameters. Let the life status of the current message of an agent i at a time instant t be denoted as $l_i(t)$. Initially, the life status of agent i is drawn from the uniform distribution $U[0, L_m]$ $\forall i$. It is understood that by choosing such an initial condition and a reasonable value of L_m, one can obtain a steady-state condition of the mobile agent network after the initial transience, where the total number of inter-agent communication links fluctuates around a constant value. Although obstacle avoidance is a natural component in any agent mobility model, it is not considered in the current setup. To analyze the present network, certain non-dimensional quantities can be constructed involving ρ, R, v and L_m (please see [13] for details). However, in real life, change of R and v for an agent may involve expensive procedures. To change ρ, either new agents need to be deployed or some agents have to be withdrawn; hence, ρ is not a suitable control parameter. Thus, L_m is considered to be the only practical tunable parameter for control purposes in the present context. The following section analyzes the effects of L_m on the time averaged network topology that is primarily represented by the expected degree and the degree distribution of the network.

B. Degree Distribution

The degree of a node is defined to be the number of nodes in the network it is connected to and the degree distribution $P(k)$ for a network is defined to be the probability distribution function of degrees over the entire network. These definitions are straight forward for static networks. In the present context of dynamic networks, let $P_i(k, L_m(i))$ be defined as the distribution (computed over time) of the number of distinct agents that communicate with a given agent (node) i within its one message lifetime $L_m(i)$. The degree distribution as a function of L_m is defined as:

$$P(k, L_m) = \frac{1}{n(L_m)} \sum_{\{i:L_m(i)=L_m\}} P_i(k, L_m(i)) \qquad (1)$$

where, $n(L_m)$ is number of agents in the network with message lifetime L_m. Finally, the overall network degree distribution can be defined as the expected value of $P(k, L_m)$, i.e.,

$$P(k) = \frac{1}{N} \sum_{\{L_m\}} n(L_m) \cdot P(k, L_m) \qquad (2)$$

where N is total number of agents in the network. As described earlier, the mobile agent network model considered in this paper follows the structure of proximity networks that is used for modeling several social network phenomena. It has been shown in literature [5], [13] that the nature of degree distributions for proximity networks can be different (e.g., Poisson distribution and Power law distribution as in scale-free networks) depending on the parameters of the mobile agent dynamics. For the parameter considerations made in this paper (described in 2-A), it is expected that the time-averaged degree distribution $P(k, L_m)$ will follow a Poisson distribution. A brief sketch of the proof is provided here for completeness of the paper.

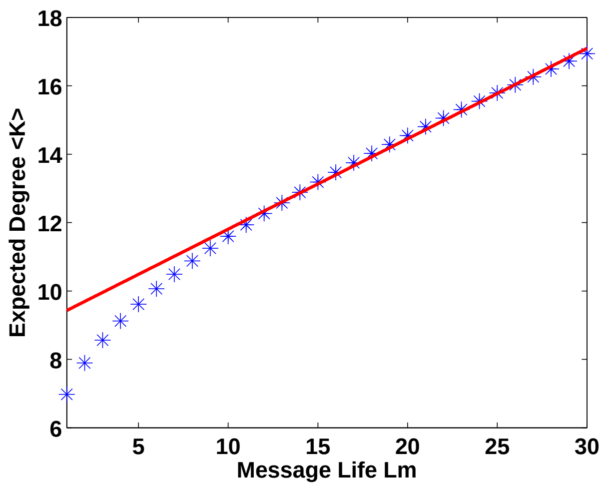

Fig. 1. Variation of Expected Degree $< k >$ of the Network with Homogeneous Message Life L_m

Let an agent i with message lifetime L_m be considered for analysis and the probability that an agent j $(j \neq i)$ comes inside the zone of communication of i within the time window L_m be denoted as $p_{ij}(L_m)$. Clearly, $p_{ij}(L_m)$ is an increasing function of L_m. Now, given any stage of network evolution, the probability that i and j communicate in the next time step is modeled as $\alpha g_i g_j (\leq 1)$, where g_l is called the *gregariousness* (i.e., tendency to communicate with other agents) of agent l in accordance with social network literature and α is a parameter that incorporate spatial information of the network, such as the agent density. Also, it is understood that g_l is a function of the radius of communication and velocity of agent l. With this model and assuming independent activity at each time step, the probability that agent i and j do not communicate within L_m is $(1 - \alpha g_i g_j)^{L_m}$. Therefore,

$$p_{ij}(L_m) = 1 - (1 - \alpha g_i g_j)^{L_m} \tag{3}$$

The expected degree $< k >_i$ can be calculated as:

$$\begin{aligned} < k >_i &= \sum_{j=1}^{N} p_{ij}(L_m) \\ &= \sum_{j=1}^{N} [1 - (1 - \alpha g_i g_j)^{L_m}] \\ &\simeq \alpha L_m g_i (\sum_j g_j) \quad for\ \alpha L_m << 1 \end{aligned} \tag{4}$$

The assumption of $\alpha L_m << 1$ is realized if α is very small and at the same time L_m does not have a very large value; small value of α provides an upper bound on the maximum number of agents that agent i can communicate in one time step (this is also known as the *exclusion constraint* [5]). In this paper, radius of communication and velocity are the same for every agent. Thus, all agents share a uniform gregariousness, say g. Therefore, $p_{ij}(L_m)$ is independent of agent specifications i and j and is denoted as $p(L_m)$ or simply p. Also, all agents are assumed to have same message lifetime L_m. With these assumptions, numerical experiments are performed to calculate the expected degree $< k >$ of the network for various values of homogeneous L_m. Figure 1 shows the result obtained from these experiments and an approximately linear relation between $< k >$ and L_m (as derived before) can be observed beyond $L_m = 9$. Now, with homogeneous p and L_m across the network, the degree distribution $P(k, L_m)$ is written as:

$$P(k, L_m) = \binom{N}{k} p^k (1 - p)^{N-k} \tag{5}$$

With a sufficiently large number of agents, i.e., $N >> 1$, the binomial distribution $P(k, L_m)$ asymptotically approaches the following Poisson distribution.

$$P(k, L_m) = \frac{< k >^k}{k!} e^{-<k>} \tag{6}$$

Figure 2 shows the plots of degree distribution $P(k, L_m)$ obtained from numerical experiments performed for $L_m = 1, 20, 30$ which confirms the above claim. Note that the degree distribution for $L_m = 1$ represents the characteristics of a static proximity network. However, it is shown in [5] that by choosing non homogeneous $p_{ij}(L_m)$, one may obtain other types of degree distributions (e.g., power-law distribution) as well. Thus, degree distribution and expected degree of the network (i.e., the expected network topology) can be controlled by varying L_m.

3. Binary Decisions with Externalities: Agent Interaction Dynamics

Many decentralized agent interaction policies are reported in literature for the purpose of decision/awareness propagation as described in Section 1. Binary decisions with externalities policy is a simple example of such algorithms.

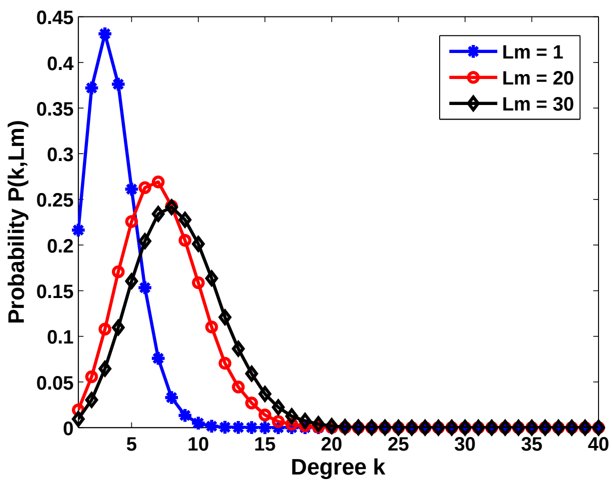

Fig. 2. Plot of Degree distribution $P(k, L_m)$ of the Mobile agent network for $L_m = 1, 20, 30$

However, it can successfully model complex real life phenomena, such as cascading failure in power grids and opinion spreading in social networks [12]. Also, this policy is suitable for mobile agent networks, where lack of homogeneous neighborhood structure is inevitable (as opposed to regular lattices, where majority vote or random-field Ising model policies may be more effective). The following subsections describe the agent interaction policy and the necessary global cascading conditions for the mobile agent network introduced before.

A. Agent Interaction Policy

The state of every agent can either be -1 or $+1$. Let an agent A observes the states of k other agents that come within its communication radius during the lifetime L_m of its one message and if A comes across one agent multiple times, then it records the latest state information of that agent. Suppose, i of such k agents are observed to be in state +1. Then the state updating policy of A is governed by a threshold parameter ϕ such that, if $\phi < i/k$, then A will assume state $+1$ for next L_m time units, otherwise it will assume state -1, irrespective of its current state. Although, in general ϕ can be heterogeneous, i.e., different ϕ for different agents, it is assumed to be uniform in this paper. It is shown in [12] that for static graphs, i) the critical value of ϕ below which global cascade occurs and ii) the expected size of the cascade depend only upon the degree distribution (expected degree or coordination number) and the distribution of ϕ among agents. Formally, to see the dynamics of global cascade, one may start with a population of agents where all are in state -1. Then states for a small number of randomly selected agents (known as seeds) are perturbed to $+1$ at time, $t = 0$. After that, the population is allowed to evolve according to the rules described above. A global cascade condition is said to have triggered if at least a threshold fraction C of the total agent population change state to $+1$. C is taken as 0.9 in this paper. Usually, in theoretical analysis (as performed in [12]), the underlying network is assumed to be sufficiently large (for example, with 10^4 agents) where as the number seeds is considered to be sufficiently small (for example, approximately three orders of magnitude less than the total number of agents). In that case, it is observed that the dependence of the cascading condition on the seed size is very small. Although, these assumptions may be relevant in social network or epidemiology studies, in the case of mobile agent engineering applications, they may not be very accurate. Hence, effects of large number of seeding agents on a relatively smaller sized network need to be studied. However, these realistic considerations make theoretical analysis more complicated and in some cases intractable.

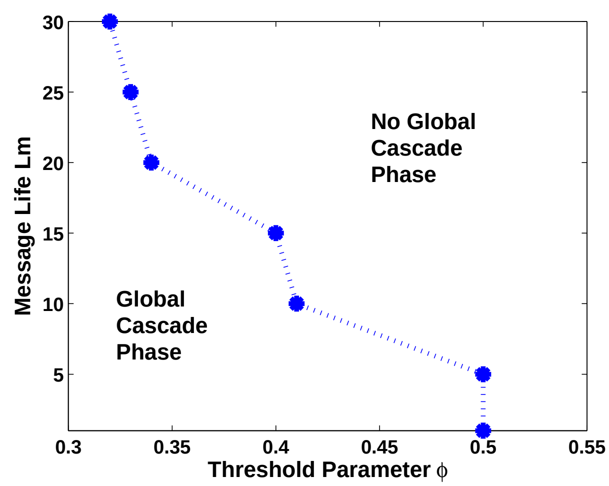

Fig. 3. Phase Diagram plot of L_m vs. ϕ for the Global Cascading phenomenon in Mobile agent networks

B. Global Cascade with Large Seed

Based on the motivation provided above, this subsection discusses the phenomenon of global cascading with large number of seeding agents. In contrast with the case of very large network with small seed, the global cascading phenomenon depends on the size of the seed when the relative seed size is high. Therefore, the phase boundary in the $< k >$ vs. ϕ phase diagram will be a function of the size of the initial seed. In the present simulation architecture, presence of a certain expected size of seed is realized in the following fashion. The agent state dynamics starts with all agents in state -1 and the agent states evolve according the rule described earlier. Suppose the desired fraction of seed is 0.02, i.e., 2% of agents should have state $+1$. To achieve the objective dynamically, a small modification is done in the agent state updating policy; after the state updating decision is taken by an agent following the threshold based neighborhood interaction rule, there remains a finite probability of 0.02, with which the agent can go against the decision made by its neighborhood influence. It should be clear that this policy modification will dynamically generate an expected seed size of 2% even when the agent state dynamics starts with all agents in state -1. However, this modification has more implications from an engineering perspective that will be explained in Section 4. Also, as L_m is considered to be the sole parameter to dictate the value of $< k >$, given the other relevant network parameters constant, an L_m vs. ϕ phase diagram is constructed here instead of the $< k >$ vs. ϕ phase diagram. Monte-Carlo simulations have been performed (with a 2% seed) to construct the phase diagram, i.e., to identify the critical ϕ values for different L_m values. For each Monte-Carlo run with specific values of L_m and ϕ, the onset of global cascading is monitored. Probability of global cascading is calculated from the observations of 50 such runs for each L_m-ϕ combination. At a given value of L_m, the critical ϕ^* is considered to be the highest value of ϕ for which the probability of global cascading is above a threshold value of 0.95. The phase diagram is shown in fig. 3 and it can be observed that it is characteristically very similar to the phase diagram for small seed case with very large static networks, shown in [12].

4. Global Awareness of a Localized Hotspot: an example application

This section presents an application of global cascade phase transition in distributed decision making and propagation of the decision. Consider the case of multiple agents performing surveillance in a given region, where the agents are tasked with detection of threats in the region. A typical example of such a threat could be plumes of harmful chemicals that have to be detected. Taking into account the nature of these threats, they may be modeled as local hotspots within the surveillance region. Only a few agents that search areas within the hotspot have a non-zero probability of detecting the threat. The aim of this section is to develop a distributed and leader-less algorithm for mobile agents that is able to decide if the threat is significant enough. Further, the algorithm should be capable of propagating the information of a significant threat to other agents that may be far off from the local hotspot. However, if the threat is not significant then agents only in the local vicinity should become aware of that. Previous literature [14] have extensively studied the gradient based approaches for detection of hotspots. These approaches primarily focus on the moving agents towards the hotspots based on distributed estimation of gradients. However, in this application, it is required is that all agents should become cognizant of the presence of the threat while operating and monitoring in their own respective local areas. In our approach, the presence of a hotspot does not affect the motion of the agents. Instead, the information states of other agents are updated to reflect the required level of awareness that the agents should posses regarding the threat. The motivation here is to disseminate information away from the local hotspot to the entire population of agents. The following subsection presents a brief description of the hotspot surveillance problem and applies the notion of global cascade phase transition to effect decision making and its propagation though out the network.

A. Problem Description

Consider the mobile multi-agent network and the interaction policy explained in Sections 2 and 3. The area of the surveillance region (A), number of mobile agents (N), radius of communication (R), displacement per unit time (x) carry the same values as before. Also, message lifetime (L_m), and the threshold parameter (ϕ) is same as defined earlier. A hotspot (i.e. a region where threat exists) is modeled as a map for probability of detection of the threat. The probability of detection, P_D is maximum at the center of the hotspot and decays to zero in a radially symmetric manner; it is characterized by two parameters: i) The maximum probability of detection of threat, P_{Dmax} ($= 0.8$ in this study) and ii) the effective radius (r_{hs}) of the circular region within which the detection probability of the threat is greater than 0.5, i.e., agents further than a distance of r_{hs} from the center of the hot-spot have less than 0.5 probability of detecting the threat. For convenience, a hotspot length scale λ is defined as the non-dimensional quantity r_{hs}/L. The two states $\{+1, -1\}$ are used to indicate the information acquired by the agents. An agent in state $+1$ is said be aware of the existence of a threat/hotspot somewhere in the surveillance regions (state of alert), whereas state -1 implies that the agent believe that there are no threats in the entire region. After the expiry of message lifetime L_m, the state of an agents may be updated based on the following rules.

- Agent state becomes $+1$ upon detecting a threat; clearly, detection depends on the proximity of the agent to the center of the hot-spot, i.e., value of P_D at its current location.
- States are updated based on the agent interaction policy (see Section 3) as characterized by the threshold parameter ϕ and message lifetime L_m.
- There remains a finite probability of 0.02 as before, with which the agent can go against the decision made by its neighborhood or hotspot influence.

These state update rules exhibit global cascades that *propagate* information from the local hotspot. Such a cascade would certainly be dependent on the size of the hotspot and this can be used to design a decision propagating mechanism that is robust to spurious noise and false alarms but responds to hotspots of significant influence. For example, only a few agents (at a time) can move very close to the center of a small hotspot and possibly change their states to $+1$. However, soon after they move out of the hotspot influence

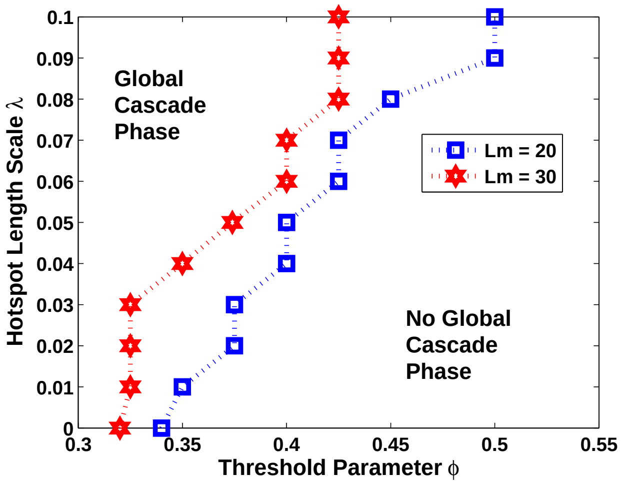

Fig. 4. Phase Diagram plot of λ vs. ϕ for the Global Cascading phenomenon in Mobile agent networks with different L_m

zone, their states revert back to -1 due to interactions with other agents that are in state -1. As a consequence, there is no decision propagation and the agents far from the hotspot do not become aware of the existence of a threat. In contrast, a large hotspot may be visited by higher number of agents at once. These agents exit the hotspot with state $+1$ and have the combined dominance to influence other agents to change their states to $+1$, thereby propagating information regarding the presence of a threat to the entire network. Thus, this application exploits the large seed effects, where the probability of occurrence of a global cascade is determined by the initial fraction of agents in state $+1$.

B. Results and Discussion

As evident from above, there exist a critical length scale λ beyond which the all the agents become cognizant of the hotspot. The choice of ϕ and L_m determines the value of this critical length scale. Since a larger value of ϕ decreases the propensity for the network to undergo a cascade, a larger hotspot is required to provide sufficient seeds to initiate the propagation. Additionally, a longer message lifetime implies a higher value of the expected degree of the network. As a consequence, a larger seed/hotspot is required for a global cascade. Figure 4 shows the λ vs. ϕ phase diagram for $L_m = 20, 30$. This phase diagram enables the user to choose values of ϕ and L_m to distributively make an implicit decision to propagate knowledge about the hotspot. Six plates in Fig. 5 show the effects of ϕ on the global cascade for a hotspot of a given length scale, $\lambda = 0.05$ on a network with $L_m = 30$. The upper three plates show the time transitions of the agent states for $\phi = 0.45$. No global cascading is observed as ϕ is too large for a hotspot of this length scale. However, if ϕ is reduced to 0.35, a global cascade may be observed, as seen in the bottom three plates of Fig. 5.

Remark 4.1: It should be noted that the global cascade phenomenon is irreversible, implying that an already alarmed network (corresponding to 90% of agents in $+1$ state) would not revert back to an unalarmed state (corresponding to 90% of agents in -1 state) on its own even when the threat is over. The address this issue, an increase of the global parameter ϕ is suggested to reset the network from alarmed to unalarmed state. The finite probability (0.02 in this case) of agents to go against the decision made by neighborhood or hotspot influence will ensure a minimum seed size for this transition.

5. Summary, Conclusions and Future work

This article addresses the distributed decision propagation problem in mobile agent networks. The mobile agent scenario is modeled as a proximity network that leads to a time-averaged Poisson degree distribution. Extensive numerical simulations have been performed to characterize the global cascading phase transition phenomenon using the binary decisions with externalities mechanism for such networks. It is found that in realistic mobile agent applications, message broadcasting duration/state updating interval can be tuned to change the effective network topology. On the other hand, the threshold parameter in the binary decision policy can be tuned to change the decision propagation nature. Also, the phase transition phenomenon is not independent of the (large) seed size and this fact has been exploited in the application example to show that the decision propagation can be triggered only beyond a user defined threshold threat level. As this is an initial work in this direction, there are many issues remain to be resolved. Among them the following are currently being pursued.

- Determining the finite size scaling laws for the network topology model and the current agent interaction policy;
- Investigating the effect of multiple significant sized hotspots;
- Investigating agent dynamics with more than two possible agent states;
- Investigating the behavior of more complex agent interaction policies, such as the Ising model, on proximity networks.

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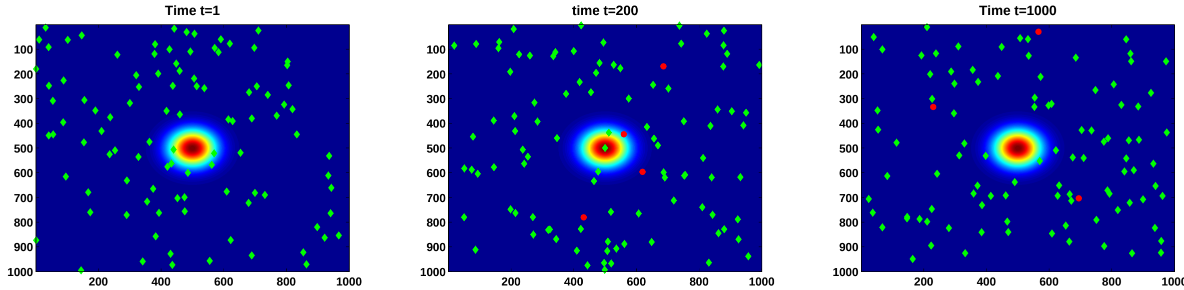
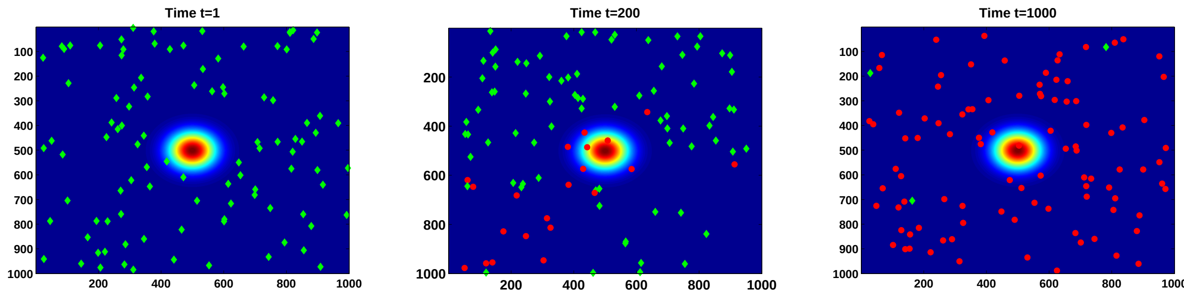


(a) Threshold Parameter, $\phi = 0.45$ (b) Threshold Parameter, $\phi = 0.45$ (c) Threshold Parameter, $\phi = 0.45$



(d) Threshold Parameter, $\phi = 0.35$ (e) Threshold Parameter, $\phi = 0.35$ (f) Threshold Parameter, $\phi = 0.35$

Fig. 5. Propagation of Global awareness for Hotspot length scale $\lambda = 0.05$ on a Mobile agent network with Message lifetime $L_m = 30$. Diamond (green in color) shape denotes agents with state -1 and Circle (red in color) shape denotes agents with state $+1$.

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