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Distributed Decision Propagation in Mobile Agent Networks

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***Abstract*—This paper develops a language-measure-theoretic distributed algorithm for decision propagation in a mobile-agent network topology. The agent interaction policy proposed here enables the control of the tradeoff between *Propagation Radius* and *Localization Gradient*. Analytical results regarding statistical moment convergence are presented and validated with simulation experiments.**

1. Introduction

Distributed information propagation and control mechanisms in mobile-agent networks have drawn much attention recently due to their relevance in engineering problems. Particularly, in a resource constrained environment, mobile agents have potential advantages over static networks in terms of coverage and time-criticality. Application areas include surveillance and reconnaissance by autonomous vehicles with limited capabilities, trust establishments in mobile ad hoc networks (MANETs) [1] and threat monitoring by mobile sensor networks. Furthermore, diffusion of aggregated information is more relevant compared to individual sensor information [2] mostly due to its robustness to individual agent's failure in detection/communication. In this context, this paper deals with global propagation of a localized awareness in a leaderless environment in a robust and completely distributed manner.

In general, there are two aspects of interacting agent systems, namely (i) network topology and (ii) agent interaction dynamics. Recently, the analysis of so called proximity networks [3] (also called the moving neighborhood networks [4]) has been performed to model contact/collision based disease spreading. In a recent paper [5], the authors used a similar approach to analyze mobile agent networks for engineering application. The mobile agent network used in this paper follows the same structure. Distributed agent interaction dynamics for decision propagation has several mechanisms available in literature, examples are game theoretic [1], biology inspired, physics inspired (Ising/Potts models) [6], bootstrap percolation [7] and majority voting [8]. Gossip algorithms are the most studied interaction dynamics in the context of consensus [9]. However, in many applications, large group of agents do not seek consensus. Often localized percolation of decision is desired to localize the information source.

The main contribution of this paper is the development of a generalized gossip algorithm based on the recently developed *language-measure theory* [10], [11] for a time-varying network topology. The algorithm is used for distributed global propagation of decision/awareness in mobile-agent networks and it is shown that the generalization parameter controls the tradeoff between *Propagation Radius* and *Localization Gradient*. Analysis of moment dynamics [12] (up to second order) is presented and verified using simulation experiments. Variance analysis is performed under two conditions: (i) *Congruous time-scale:* when network topology evolution and agent interaction dynamics has similar time-scales and (ii) *Disparate time-scale:* when (faster) agent interaction dynamics can be considered as a singular perturbation with respect to (slower) network topology evolution.

2. Formulation of the Problem

This section describes a mobile-agent scenario in terms of proximity networks [3] and subsequently formulates the agent interaction policy.

A. Model Description

Let the area of a two dimensional (Euclidean) operational region be A. In the present case, it is assumed to be a square area with side length L, i.e., $A = L^2$. Initially, N agents are distributed randomly in the given area, and the agent density is defined as $\rho = N/A$. The uniform radius of communication for each agent is denoted by R, i.e., two agents can only communicate (to say, exchange their state values) when the distance between them is less than R. The agents move in a 2-D random walk where the speed v is same for all the agents in the current setup. The random walk is realized by independently choosing a direction of motion from a uniform distribution $U(0, 2\pi)$, by all agents at every time step. During its motion, every agent is assumed to broadcast a message (e.g., its own state) over a certain time window that is known as the message lifetime L_m. At the same time, the agent receives messages (e.g., state values) from other agents, which may come within the distance R. After expiry of a message, an agent possibly undergoes a state updating and then it starts broadcasting the new state for another window

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of message lifetime. If the message lifetime L_m is very small, then the effective network will be equivalent to multiple instances of static random graphs. On the other hand, the network eventually becomes fully connected as $L_m \to \infty$. Thus, to model realistic situations, L_m should be chosen appropriately based on the other network parameters. Although the message/state updating may occur in a non-synchronous manner in the agent population, for analytical tractability of the agent interaction policy, only synchronous message/state updating is considered in this paper. Furthermore, obstacle avoidance is a natural component in any agent mobility model and it is not considered in the current setup. In this setup, the degree of a node (agent) is defined to be the number of distinct nodes (agents) in the network it connects (communicates) to within a message lifetime L_m. The above setup is only used for simulation study of the paper. Therefore, please see [5] for detailed discussion on nature of expected degree distribution and expected degree of the network class considered here. Furthermore, note that a slower time-scale corresponding to measure updating can be considered compared to the fast time-scale of agent motion. Let, time in this slower scale be denoted by τ. The agent interaction policy is introduced in the sequel.

B. *Agent Interaction Policy*

The generalized gossip interaction policy developed in this paper is essentially based on the concepts of signed real measure of probabilistic regular languages generated by probabilistic finite state automata (PFSA) [10], [11]. However, the details are omitted for simplicity and the policy is presented here in a self-sufficient way.

Consider the case of multiple agents performing surveillance, where the agents are tasked with detection of threats in a given region. A typical example of such a threat could be plumes of harmful chemicals that have to be detected. Taking into account the nature of these threats, they may be modeled as local hotspots within the surveillance region. Only a few agents that search areas within the hotspot have a non-zero probability of detecting the threat. The aim of this paper is to develop a distributed and leader-less algorithm for mobile agents that is able to disseminate the information of a threat to other agents that may be far off from the local hotspot in a controlled way. Previous literature [13] have extensively studied the gradient based approaches for detection of hotspots. These approaches primarily focus on the moving agents towards the hotspots based on distributed estimation of gradients. However, in this application, it is required that all agents should become cognizant of the presence of the threat while operating and monitoring in their own respective local areas. In our approach, the presence of a hotspot does not affect the motion of the agents. Instead, the information states of other agents are updated to reflect the required level of awareness that the agents should posses regarding the threat. The motivation here is to disseminate information away from the local hotspot to the entire population of agents. A language-measure-theoretic approach to this problem is developed in the sequel.

Problem Setup: Let the connectivity graph be denoted as G and the corresponding adjacency matrix be denoted as A. The adjacency matrix of the mobile agent network (described in Section 2-A), after the expiry of the message lifetime L_m, will have ones in the ij^{th} position if agent i communicates with agent j in the period of L_m, otherwise the matrix element will be zero. All the diagonal elements of the adjacency matrix are kept as zeros. The Laplacian matrix ($\mathcal{L}$) of G is defined as: $\mathcal{L} = D - A$, where, D denotes the degree matrix. Degree matrix D is defined as the diagonal matrix with $D_{ii} = d_i$, where d_i is the degree of node i. Finally, the agent interaction matrix Π is considered as [14]:

$$\Pi = I - \beta\mathcal{L} \tag{1}$$

The parameter β is chosen appropriately such that Π becomes a stochastic matrix and its second largest eigenvalue satisfies the condition $|\lambda_2(\Pi)| < 1$. In the context of Proximity networks, this requirement of keeping Π stochastic is achieved by taking $\beta = \frac{1}{d_{max}+1}$. Now, β is a parameter that is chosen off-line and hence so is d_{max}. Therefore to satisfy the above condition on-line, an agent ignores communications with distinct agents beyond d_{max} number of agents within a message lifetime L_m (note, d_{max} is pre-determined). However, given the degree distribution $P(k, L_m)$ (for a fixed message lifetime L_m), d_{max} is chosen to be large enough such that the probability of $k > d_{max}$, is very low. For simulation studies, the low probability is taken as $\epsilon = 0.001$. Note that Π is a stochastic and symmetric (i.e., also doubly stochastic) matrix due to the above construction procedure.

In the present context, hotspots are detected only by agents proximal to them. From this perspective, agents (nodes) have a state characteristic function $\chi : Q \to \{0, 1\}$, where Q denotes the set of nodes. $\chi^i = 1$ means that the agent i has detected a hotspot and $\chi^i = 0$ denotes otherwise. Agents (nodes) also have real measure values $\nu : Q \to [0, 1]$. In this context, a higher value of ν^i denotes a higher level of awareness of agent i about a hotspot in the operational area.

Decentralized Strategy: The decentralized strategy proposed here involves synchronous updating of measures of all agents after the expiry of one message lifetime, L_m. Naturally, L_m is also homogeneous in the agent population. Correspondingly, if an agent i detects a hotspot at some instant, $\chi^i = 1$ is kept till the next global measure updating even if the agent does not see the hotspot anymore. Thus, it is clear that both ν and χ are functions of τ (slow time-scale of measure updating, as defined in Section 2-A). According to the definition provided above, the matrix Π is also a function of τ.

With this setup, a decentralized strategy for measure updating in the mobile-agent population is introduced below that involves a user defined control parameter θ.

$$\nu_\theta^i|_{\tau+1} = (1-\theta) \sum_{j \in \{i\} \cup Nb(i)} \Pi_{ij}|_\tau \nu_\theta^j|_\tau + \theta \chi^i|_\tau \tag{2}$$

where, $Nb(i)$ denotes the set of agents in the neighborhood of agent i i.e., agents that communicate with agent i between τ and $\tau + 1$. Expansion of the above equation gives:

$$\nu_\theta^i|_{\tau+1} = (1-\theta) \left[(1-\beta d_i)\nu_\theta^i|_\tau + \sum_{j \in Nb(i)} \beta \nu_\theta^j|_\tau \right] + \theta \chi^i|_\tau \tag{3}$$

The above equation signifies that the self-influence for an agent reduces with increase of its degree. In other words, the more neighbors an agent communicates to, the less it relies just on its own observation. Vectorially the dynamics can be expressed as:

$$\nu_\theta|_{\tau+1} = (1-\theta)\Pi|_\tau \nu_\theta|_\tau + \theta\chi|_\tau \tag{4}$$

The recursive relation in the Eqn. (4) above is expanded as:

$$\begin{aligned}\nu_\theta|_{\tau+1} &= (1-\theta)^{\tau+1}[\Pi|_\tau\Pi|_{\tau-1}\cdots\Pi|_0]\nu_\theta|_0 + \theta\chi|_\tau \\ &+\theta(1-\theta)\Pi|_\tau\chi|_{\tau-1} + \theta(1-\theta)^2\Pi|_\tau\Pi|_{\tau-1}\chi|_{\tau-2} \\ &+\cdots+\theta(1-\theta)^\tau\Pi|_\tau\Pi|_{\tau-1}\cdots\Pi|_1\chi|_0\end{aligned} \tag{5}$$

Thus, this policy is simply a gossip algorithm with varying input $\chi|_\tau$ and varying network topology represented by $\Pi|_\tau$. The memory of a past input fades as a function of the parameter θ. Due to this notion, the above policy can be called a *generalized gossip algorithm* with θ as the generalizing parameter.

3. Convergence of Statistical Moments

The convergence results presented here involve expected quantities due to the inherent stochastic nature of the problem. Thus even in steady state, ν_θ will always fluctuate in the slow time-scale due to the fluctuation in Π and χ. However, interesting observations regarding slow time-scale evolution of the system can be made in terms of statistical moments of ν_θ computed over the agent population. In this paper, both average (over agents), $\mathbb{M}_a[\cdot]$ and variance (over agents), $\mathbb{V}_a[\cdot]$ of ν_θ are considered at steady state. Note, $\nu_\theta|_\tau$ at any slow time instant τ is an N-dimensional vector, where N is the number of agents in the population. Hence, $\mathbb{M}_a[\nu_\theta|_\tau]$ and $\mathbb{V}_a[\nu_\theta|_\tau]$ are respectively scalar average and variance values where $\nu_\theta|_\tau$ is considered as a random variable with N samples. In general, the functions $\mathbb{M}_a[\cdot]$ and $\mathbb{V}_a[\cdot]$ defined on an N dimensional column vector $\mathbf{x} = [x_1, x_2, ..., x_N]^T$ as follows:

$$\mathbb{M}_a(\mathbf{x}) = \frac{1}{N}\mathbf{1}\mathbf{x} = \mathbf{x}^{avg} \tag{6}$$

where, $\mathbf{1}$ is a row vector with all elements as 1. After mean subtraction, let the resulting vector be denoted as $\tilde{\mathbf{x}}$, i.e., $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}^{avg}\mathbf{1}^T$. Therefore, $\mathbb{V}_a(\mathbf{x}) = \frac{1}{N}\tilde{\mathbf{x}}^T\tilde{\mathbf{x}}$.

A. Convergence of Measure Average over Agents

Recall the system dynamics as given in Eqn. 4.

$$\nu_\theta|_{\tau+1} = (1-\theta)\Pi|_\tau \nu_\theta|_\tau + \theta\chi|_\tau \tag{7}$$

The following equation is obtained by pre-multiplying $\frac{1}{N}\mathbf{1}$ on both sides of Eqn. 7.,

$$\nu_\theta^{avg}|_{\tau+1} = (1-\theta)\nu_\theta^{avg}|_\tau + \theta\chi^{avg}|_\tau \tag{8}$$

Note, $\mathbf{1}\Pi|_\tau = \mathbf{1}$, as $\Pi|_\tau$ is doubly stochastic. Expanding Eqn. 8, one obtains

$$\begin{aligned}\nu_\theta^{avg}|_{\tau+1} &= (1-\theta)^{\tau+1}\nu_\theta^{avg}|_0 + \theta\chi^{avg}|_\tau \\ &+\theta(1-\theta)\chi^{avg}|_{\tau-1} + \theta(1-\theta)^2\chi^{avg}|_{\tau-2} \\ &+\cdots+\theta(1-\theta)^\tau\chi^{avg}|_0\end{aligned} \tag{9}$$

Considering the unrestricted 2-D random motion of the agents in the entire region, the process is assumed to be ergodic. Therefore, the ensemble expectation of $\chi^{avg}|_k$ can be denoted as $E[\chi^{avg}]\ \forall k$ (no time dependency). In this case, $E[\chi^{avg}]$ signifies the fraction of agents that visit hotspot(s) on average. Therefore, it is evident that with a constant strength of hotspot(s), $E[\chi^{avg}]$ remains constant over time. Taking (ensemble) expectation on both sides of Eqn. 9, the following relation is obtained at steady state (as $\tau \to \infty$).

$$\begin{aligned}E[\nu_\theta^{avg}|_\infty] &= \theta[1+(1-\theta)+(1-\theta)^2+\cdots]E[\chi^{avg}] \\ &= \theta[1-(1-\theta)]^{-1}E[\chi^{avg}] \\ &= E[\chi^{avg}] \quad \text{for } \theta \in (0,1)\end{aligned} \tag{10}$$

Therefore, using the notation of steady-state average (over agents) introduced before, the steady-state expected measure average (over agents) is obtained as:

$$E[\mathbb{M}_a(\nu_\theta)] = E[\mathbb{M}_a(\chi)] \tag{11}$$

Convergence of average measure to average χ implies that at steady state, sum of χ values over agents is same as the sum of ν values over agents. In general, the physical significance is that the detection decision of hotspot by few agents is getting redistributed as awareness over a (possibly) larger number of agents, where the total awareness measure is conserved. From this perspective, it is interesting to know the nature of measure distribution in the population and measure variance (over agents) provides an insight in this aspect. For example, an extreme case would be when measure variance is zero, that is all agents have the same measure value and it is equal to the average measure value of the population. In literature, this scenario is known as *consensus*. The opposite extreme case would be when there is no awareness propagation; only those agents have nonzero measure values that have detected hotspot(s) (i.e., have nonzero χ values). The measure variance will be equal to the variance of χ in this case and the hotspot(s) can be localized very well following the measure distribution due to a sharp localization gradient. Thus, measure distribution essentially dictates a tradeoff between *Propagation Radius* and *Localization Gradient* and variance of ν over agents quantifies the position of the system in this tradeoff scale.

B. Convergence of Measure Variance over Agents

It is evident from the discussion till now that there exists two fundamentally different time-scales, one related to network evolution and the other related to agent state dynamics. The analytical developments in the sequel will be presented for two special cases of relations between these two time-scales, namely: (i) Congruous Time-Scale (CTS) case and (ii) Disparate Time-Scale (DTS) case.

Congruous Time-Scale case: The two time-scales are equivalent in this case, which means at each slow-time epoch τ (when the agent measures are updated), the system has an independent state transition matrix Π as well as an independent state characteristic vector χ. More formally, the following *assumptions* are made under the CTS case.

- $\Pi|_i$ and $\chi|_k$ are independent $\forall i, k$.

- $\Pi|_i$ and $\Pi|_j$ are independent $\forall i, j$.
- $\chi|_i$ and $\chi|_j$ are independent $\forall i, j$.

Note, the first two assumptions are feasible under fairly general conditions whereas the third one requires a special condition of agent mobility (agents moving fast enough).

Disparate Time-Scale case: In this case, the two time-scales are very different such that, the network evolution (the slow dynamics) and the agent state updating (the fast dynamics) can be treated independently as it is done in the *Singular Perturbation theory*. This leads to the assumption that Π and χ remain time-invariant over the course of transience in agent state dynamics, i.e., agent measures converge before there is a change in Π and χ.

For variance calculation, consider post-multiplication of $\mathbf{1}^T$ on both sides of Eqn. 8,

$$\begin{aligned} \nu_\theta^{avg}|_{\tau+1}\mathbf{1}^T &= (1-\theta)\nu_\theta^{avg}|_\tau\mathbf{1}^T + \theta\chi^{avg}|_\tau\mathbf{1}^T \\ \Rightarrow \nu_\theta^{avg}|_{\tau+1}\mathbf{1}^T &= (1-\theta)\nu_\theta^{avg}|_\tau\Pi|_\tau\mathbf{1}^T + \theta\chi^{avg}|_\tau\mathbf{1}^T \end{aligned} \quad (12)$$

The above equation presents the mean dynamics for the system. Now, the following equation is obtained by subtracting the mean dynamics (in Eqn. 12) from the system equation (in Eqn. 7).

$$\tilde{\nu}_\theta|_{\tau+1} = (1-\theta)\Pi|_\tau\tilde{\nu}_\theta|_\tau + \theta\tilde{\chi}|_\tau \quad (13)$$

For calculation of variance (over agents),

$$\begin{aligned} (\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1}) &= (1-\theta)^2(\tilde{\nu}_\theta|_\tau)^T(\Pi|_\tau)^T(\Pi|_\tau)(\tilde{\nu}_\theta|_\tau) \\ &+\theta^2(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau) + 2\theta(1-\theta)(\tilde{\nu}_\theta|_\tau)^T(\Pi|_\tau)^T(\tilde{\chi}|_\tau) \end{aligned} \quad (14)$$

Taking ensemble expectation (given $\tilde{\nu}_\theta|_\tau$) on both sides (under the CTS assumptions),

$$\begin{aligned} E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})|\tilde{\nu}_\theta|_\tau] &= \\ (1-\theta)^2(\tilde{\nu}_\theta|_\tau)^T E[(\Pi|_\tau)^T(\Pi|_\tau)](\tilde{\nu}_\theta|_\tau) &+ \\ \theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] + 2\theta(1-\theta)(\tilde{\nu}_\theta|_\tau)^T E[(\Pi|_\tau)^T]E[(\tilde{\chi}|_\tau)] \end{aligned} \quad (15)$$

Since all the agents perform a random walk motion, they are equally likely to visit the hot spot. This implies that $E[(\tilde{\chi}|_\tau)] = 0$. Furthermore,

$$(1-\theta)^2(\tilde{\nu}_\theta|_\tau)^T E[(\Pi|_\tau)^T(\Pi|_\tau)](\tilde{\nu}_\theta|_\tau) \geq 0 \quad (16)$$

Therefore, for the lower bound

$$\begin{aligned} E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})|\tilde{\nu}_\theta|_\tau] &\geq \theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] \\ \Rightarrow E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})] &\geq \theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] \end{aligned} \quad (17)$$

The expected (steady-state) variance can be expressed as: $E\left[\mathbb{V}_a[\nu_\theta]\right] = \frac{1}{N}E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})]$. Using similar notation for χ, one has:

$$\frac{E\left[\mathbb{V}_a[\nu_\theta]\right]}{E\left[\mathbb{V}_a[\chi]\right]} \geq \theta^2 \quad (18)$$

Note, by construction $\tilde{\nu}_\theta|_\tau \perp \mathbf{1}^T$ [15]. Also, $\mathbf{1}$ is the stationary vector (left eigenvector corresponding to the unity eigenvalue) of a doubly stochastic matrix. Therefore,

$$(\tilde{\nu}_\theta|_\tau)^T E[(\Pi|_\tau)^T(\Pi|_\tau)](\tilde{\nu}_\theta|_\tau) \leq \Lambda_2(\tilde{\nu}_\theta|_\tau)^T(\tilde{\nu}_\theta|_\tau) \quad (19)$$

where, $\Lambda_2 = \lambda_2\left(E[(\Pi|_\tau)^T(\Pi|_\tau)]\right)$. Therefore, for the upper bound

$$\begin{aligned} E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})|\tilde{\nu}_\theta|_\tau] &\leq (1-\theta)^2\Lambda_2(\tilde{\nu}_\theta|_\tau)^T(\tilde{\nu}_\theta|_\tau) \\ &+\theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] \\ \Rightarrow E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})] &\leq (1-\theta)^2\Lambda_2 E[(\tilde{\nu}_\theta|_\tau)^T(\tilde{\nu}_\theta|_\tau)] \\ &+\theta^2 E[(\tilde{\chi}|_\tau)^T(\tilde{\chi}|_\tau)] \end{aligned} \quad (20)$$

At steady-state, $E\left[\mathbb{V}_a[\nu_\theta]\right] = \frac{1}{N}E[(\tilde{\nu}_\theta|_{\tau+1})^T(\tilde{\nu}_\theta|_{\tau+1})] = \frac{1}{N}E[(\tilde{\nu}_\theta|_\tau)^T(\tilde{\nu}_\theta|_\tau)]$. Therefore,

$$\begin{aligned} E\left[\mathbb{V}_a[\nu_\theta]\right]\left[1-(1-\theta)^2\Lambda_2\right] &\leq \theta^2\mathbb{V}_a[\chi] \\ \Rightarrow \frac{E\left[\mathbb{V}_a[\nu_\theta]\right]}{E\left[\mathbb{V}_a[\chi]\right]} &\leq \frac{\theta^2}{1-(1-\theta)^2\Lambda_2} \end{aligned} \quad (21)$$

Note, $\theta \in (0, 1]$ and $\Lambda_2 \in [0, 1]$. Figure 1 presents the plot

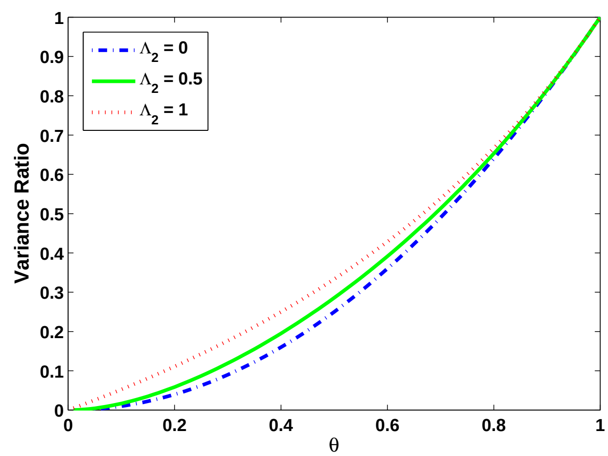

Fig. 1. Upper bounds of the variance ratio $\frac{E[\mathbb{V}_a[\nu_\theta]]}{E[\mathbb{V}_a[\chi]]}$ as a function of θ and $\Pi|_\tau$ under CTS assumptions; Lower bound is independent of Λ_2 and coincides with the upper bound for $\Lambda_2 = 0$

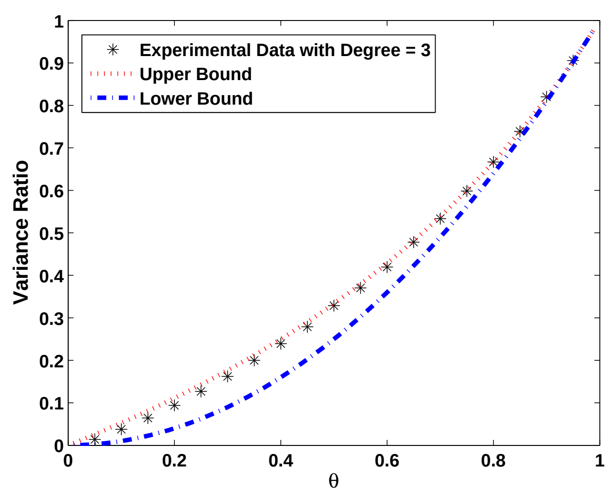

Fig. 2. Experimental Verification of Bounds on Variance Ratio $\frac{E[\mathbb{V}_a[\nu_\theta]]}{E[\mathbb{V}_a[\chi]]}$ under CTS assumptions

of upper bounds of the variance ratio $\frac{E[\mathbb{V}_a[\nu_\theta]]}{E[\mathbb{V}_a[\chi]]}$ with θ for three possible values of Λ_2. Note that the lower bound of the variance ratio is independent of Λ_2 and coincides with the upper bound for $\Lambda_2 = 0$. An experimental verification is also presented in fig. 2 that shows the experimental data to closely follow the upper bound for the particular case. While the expected degree of the network is kept as 3, high speed is assumed for agents to achieve the conditions described in the CTS assumptions.

Next, the analysis is performed under the DTS assumptions. As discussed earlier, Π and χ are not functions of τ in this case. From Eqn. 5, as $\tau \to \infty$, one has:

$$\nu_\theta|_\infty = \theta\chi + \theta(1-\theta)\Pi\chi + \theta(1-\theta)^2\Pi^2\chi + \theta(1-\theta)^3\Pi^3\chi \cdots \tag{22}$$

The following equation is obtained by subtracting the mean dynamics from Eqn. 22.

$$\tilde{\nu}_\theta|_\infty = \theta\tilde{\chi} + \theta(1-\theta)\Pi\tilde{\chi} + \theta(1-\theta)^2\Pi^2\tilde{\chi} + \theta(1-\theta)^3\Pi^3\tilde{\chi} \cdots \tag{23}$$

Using the above equation, the measure variance over agents is calculated as:

$$N\mathbb{V}_a[\nu_\theta] = \theta^2\tilde{\chi}^T\tilde{\chi} + \theta^2(1-\theta)\tilde{\chi}^T\Pi\tilde{\chi} + \theta^2(1-\theta)\tilde{\chi}^T\Pi^T\tilde{\chi} + \theta^2(1-\theta)^2\tilde{\chi}^T\Pi^T\Pi\tilde{\chi} + \theta^2(1-\theta)^2\tilde{\chi}^T\Pi^2\tilde{\chi} + \theta^2(1-\theta)^2\tilde{\chi}^T(\Pi^2)^T\tilde{\chi} \cdots \tag{24}$$

As Π is symmetric, one has:

$$N\mathbb{V}_a[\nu_\theta] = \theta^2\tilde{\chi}^T\tilde{\chi} + 2\theta^2(1-\theta)\tilde{\chi}^T\Pi\tilde{\chi} + 3\theta^2(1-\theta)^2\tilde{\chi}^T\Pi^2\tilde{\chi} \cdots \tag{25}$$

Since Π^k s are positive definite for $k \in \mathbb{N}$, the lower bound is obtained as

$$\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]} \geq \theta^2 \tag{26}$$

Using the same logic as before, it is evident that $\tilde{\chi}^T\Pi^k\tilde{\chi} \leq \lambda_2(\Pi^k)\tilde{\chi}^T\tilde{\chi}$ for $k \in \mathbb{N}$. Also, $\lambda_2(\Pi^k) = \lambda_2^k(\Pi)$ and $\lambda_2(\Pi)$ is denoted simply as λ_2 in the sequel. Therefore,

$$\mathbb{V}_a[\nu_\theta] \leq \theta^2\mathbb{V}_a[\chi] + 2\theta^2(1-\theta)\lambda_2\mathbb{V}_a[\chi] + 3\theta^2(1-\theta)^2\lambda_2^2\mathbb{V}_a[\chi] \cdots \tag{27}$$

By calculating the infinite sum, the upper bound is obtained as

$$\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]} \leq \frac{\theta^2}{[1-(1-\theta)\lambda_2]^2} \tag{28}$$

Note, $\theta \in (0,1]$ and $\lambda_2 \in [0,1]$. The upper bound for the variance ratio calculated above is valid for a particular Π. Figure 3 presents the plot of upper bounds of the variance ratio $\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]}$ with θ for three possible values of λ_2. Note that the lower bound of the variance ratio is independent of λ_2 and coincides with the upper bound for $\lambda_2 = 0$. Experimental verification is presented in fig. 4 that shows the experimental data for two cases with expected degree of the network as 3 and 7. Agent speed is kept very low (but not zero) to achieve the conditions described in the DTS assumptions.

It is observed in both cases that upper bound and lower bound coincide as θ approaches extreme values, 0 or 1 and

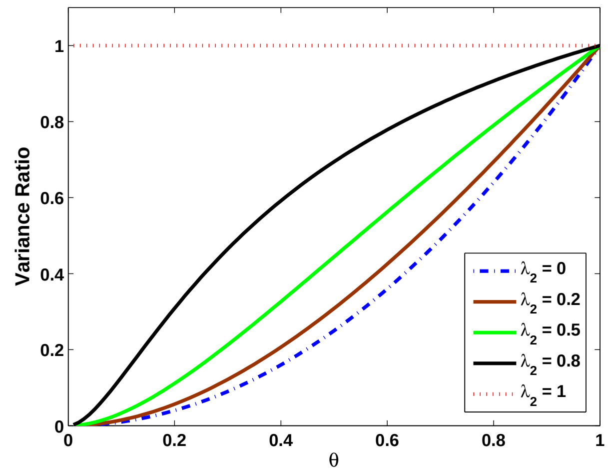

Fig. 3. Upper bounds of the variance ratio $\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]}$ as a function of θ and Π under DTS assumptions; Lower bound is independent of λ_2 and coincides with the upper bound for $\lambda_2 = 0$

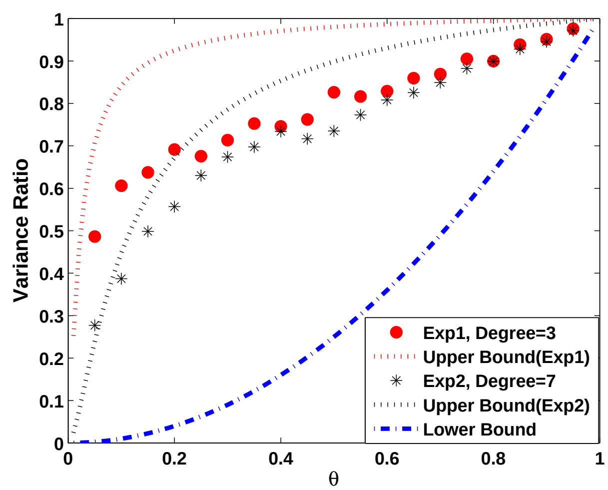

Fig. 4. Experimental Verification of Bounds on Variance Ratio $\frac{\mathbb{V}_a[\nu_\theta]}{\mathbb{V}_a[\chi]}$ under DTS assumptions

$\mathbb{V}_a[\nu_\theta] \to 0$ as $\theta \to 0$ and $\mathbb{V}_a[\nu_\theta] \to \mathbb{V}_a[\chi]$ as $\theta \to 1$. In other words, the agent population approaches *consensus* as $\theta \to 0$. In this case, although the entire population becomes aware of the hotspot(s), there is no localization gradient as every agent has same measure value. On the other hand, with $\theta \to 1$, the localization gradient improves at the cost of propagation radius. In general, $\mathbb{V}_a[\nu_\theta]$ decreases with decrease in θ. The other system component affecting the variance ratio is the Π matrix. In both CTS and DTS cases, this effect is realized through second largest eigenvalue of Π. Lower second largest eigenvalue of Π signifies more connectivity among agents. This fact explains the decrease in variance ratio with decrease in the second largest eigenvalue. It is evident that second largest eigenvalue is a function of degree of the network. However, degree certainly is not the only network parameter that determines the decision propagation characteristics as it is observed in the experimental data for CTS and DTS cases. Therefore, apart from network degree, compatibility between time-scales of network evolution and agent state dynamics also plays a key role in determining the system characteristics.

4. Summary, Conclusions and Future Work

This paper addresses the problem of distributed decision propagation in a mobile-agent network environment for surveillance and reconnaissance. A generalized gossip algorithm derived from the concepts of recently developed language-measure-theory is used for modeling the agent interaction dynamics. A completely decentralized implementation of this algorithm is shown to be useful for propagation of global awareness regarding a local hotspot in the operational area. Analytical results have been obtained for convergence of measure (awareness level) distribution in the agent population. A (user-defined) critical parameter θ, that controls the tradeoff between the propagation radius and the localization gradient. In this setting, *consensus* can be achieved as $\theta \to 0$. Two cases (CTS and DTS) relating the time-scales network topology and agent interaction are presented and verified by numerical simulation. In this algorithm, the system resets automatically upon removal of a hotspot. Another advantage of this approach is that it extends to multiple hotspot scenarios; future work will involve detailed investigation with multiple hotspots. Following are the other future research directions that are currently being pursued:

- Analytical evaluation of the expected characteristics of Π (hence, second largest eigenvalues), given the expected characteristics of the proximity network;
- Investigation of scenarios with asynchronous measure updating and heterogeneous message lifetime distribution;
- Relaxation of assumptions for variance calculation and identification of the network size-scaling laws;
- Analysis of convergence dynamics/time under generalized gossip framework.

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