Multimodal Spatiotemporal Information Fusion using Neural-Symbolic Modeling for Early Detection of Combustion Instabilities

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***Abstract*— Detection and prediction of combustion instabilities are of interest to the gas turbine engine community with many practical applications. This paper presents a dynamic data-driven approach to accurately detect precursors to the combustion instability phenomena. In particular, grey-scale images of combustion flames have been used in combination with pressure time-series data for information fusion to detect and predict flame instabilities in the combustion process. These grey-scale images are analyzed using deep belief network (DBN). The cross-dependencies between the features extracted by the DBN and the symbolic sequences generated from pressure time-series are then analyzed using $\times D$-Markov (pronounced *cross* D-Markov) models that are constructed by a combination of state-splitting and cross-entropy rate; this leads to the development of a variable-memory cross-model as a representation of the underlying physical process. These cross-models are then used for detection and prediction of combustion instability phenomena. The proposed concept is validated on experimental data collected from a laboratory-scale swirl-stabilized combustor apparatus, where the instability phenomena are induced by typical protocols leading to unstable flames.**

I. Introduction

Strict emission regulation has initiated a paradigm shift in the nominal operating conditions of gas turbine engines. Consequently, the technology of gas turbine engines has gradually adapted to low equivalence ratio combustion to suppress emissions of nitrogen oxides (NOx), instead of combustion at near-stoichiometric conditions. Ultra-lean premixed and pre-vaporized combustors, while being environment-friendly, is susceptible to combustion instabilities that are typically characterized by pressure waves with sharp tones and high amplitudes. The complexity of such an instability problem accrues from the mutual interactions among the unsteady heat release rate, flow fields and acoustics, which outline the general features of combustion instability [1], [2], [3]. Combustion instabilities have many detrimental effects on flight-propulsion dynamics and structural integrity of gas turbine engines. In this paper, we propose a data-driven approach to detect precursors to instability of the combustion process using sequential hi-speed images and pressure time-series. These two types of data are pre-processed and then combined together as a Probabilistic Finite State Automaton (PFSA) within the framework of $\times D$-Markov machines [4], [5], [6].

The complexity of combustion instability phenomena has drawn a lot of attention in the research community, which has resulted in a broad spectrum of activities ranging from model-based to data-driven analyses. Several reduced-order modeling-based approaches are presented in [7], [8]. Most of these works emphasize understanding of the system stability by means of solving the dispersion relation. More recently, a swirl combustor has been characterized and a wide range of experiments relating swirl flows. The presence of precessing vortex core (PVC) as the dominant coherent structure has been reported, where nonlinear interactions between heat release rate oscillations and PVC as the cause of superposed frequencies in time series data has been pointed out [9]. Much of the literature is dedicated to detection and correlation of these coherent structures to heat release rate and unsteady pressure oscillations. Popular methods for detection of coherent structures include proper orthogonal decomposition (POD) [10] and dynamic mode decomposition (DMD) [11], which use tools of spectral theory to derive spatial coherent structure modes. Specifically, DMD has been used to estimate the growth rates and frequencies from experimental data and also for stability analysis of the experimental data. More recently, a deep-learning [12] and symbolic time series analysis (STSA) [13]-based approach has been presented in [14] for early detection of combustion instabilities. Recently, detection of lean blowout in combustors have also been reported in [15], [16].

This paper presents a sensor-fusion-based approach for early detection of instability phenomena in combustors. Sensors of different modalities (e.g., hi-speed grey-scale images and pressure time-series data) have been used to observe the same physical process that can capture the expected outcome of one modality based on the state of the other modality. To develop cross-models for multimodal sensor fusion, first a deep belief network (DBN) [17] is used to process the hi-speed grey-scale images for dimensionality reduction and automatically learning the features from the combustion images. Then, the cross-models of the features from images and pressure time-series data are represented as a $\times D$-Markov machine [4], [5], [6] that is capable of capturing the behavior of a symbolic process conditioned on another symbolic process. These $\times D$-Markov machine models are then used to detect and predict combustion instabilities. The proposed approach offers the benefit that data from two different sensor modalities can be used to arrive

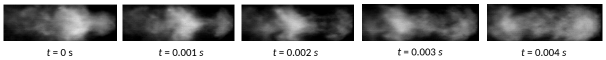


Fig. 1: Visible coherent structure in greyscale images at $Re = 15,942$ and full premixing for a fuel flow rate of 0.495 g/s

at more robust and accurate prediction of the underlying physical process, and its efficacy has been validated using experimental data of combustion from a laboratory-scale swirl combustor apparatus [18]; in particular, hi-speed grey-scale images and pressure time-series data have been used for detection and prediction of combustion instabilities in a laboratory-scale swirl combustor apparatus [18]. To the best of authors' knowledge, such a symbolic causal modeling-based approach of images with time-series data has not been proposed before in open literature.

II. Experimental Description

The experimental apparatus is a laboratory-scale combustor with a swirler of diameter 30 mm with 60 degree vane angles (i.e., geometric swirl number of 1.28). Air is fed into the combustor through a settling chamber of diameter 280 mm with a sudden contraction leading to a square cross section of side 60 mm, providing an acoustically open condition with area ratio of 17. A mesh and honeycomb structure at the immediate downstream of the contraction assures uniform flow to the swirler. The combustorconsists of a 200 mm long inlet section, an inlet optical access module (IOAM) of length 100 mm, a primary combustion chamber of length 370 mm, and secondary duct of the same length. The overall length of the constant area ducts was chosen to be 1340 mm. The fuel injection tube is coaxial to a mixing tube which has the same diameter as that of the swirler. The bypass air that does not enter the mixing tube passes through slots on the swirl plate. The slots on the fuel injection tube are drilled at designated distance upstream of the swirler, which dictates the extent of premixing between fuel and air. The larger this distance, more homogeneous the air-fuel mixture is. Two upstream distances of $90mm$ and $120mm$ were chosen for fuel injections during the experiments, where the former of the two denotes partial premixing and the later provides full premixing. The hi-speed images were collected through IOAM at 3 kHz using Photron High speed star with a spatial resolution of 1024×1024 pixels. Synchronized pressure data was acquired using piezoelectric transducers (PCB make) with resolution 225 mV/ kPa at a location downstream of the IOAM. The data acquisition was triggered simultaneously using NI card and taken for a duration of $3s$ yielding in a sequence of 9,000 images for every operating condition. More details of the combustor and the performed experiments can be found in [14].

Figure 1 presents sequences of images of dimension 392×1000 pixels for unstable ($Re = 15,942$, $FFR = 0.495g/s$ and full premixing) state. The flame inlet is on the right side of each image and the flame flows downstream to the left. As the combustion is unstable, coherent structure formation along the flow is observed. Figure 1 shows formation of mushroom-shaped vortex at $t = 0, 0.001s$ and the shedding of that towards downstream from $t = 0.002s$ to $t = 0.004s$.

III. Information Fusion Framework and Tools

This section delineates the proposed framework for early detection of combustion instability via fusing multi-modal data arriving from hi-speed camera and pressure transducer that are spatially apart. In the framework of analysis adopted in this paper, a deep belief network (DBN) is followed by a symbolic time series analysis (STSA) module. During training, hi-speed images from both stable and unstable states for various operating conditions are used as the visible layer V of a DBN. Multiple hidden layers (i.e., h_1 to h_n) with reducing dimensions [19] are cascaded after the visible layer.The weights (i.e., W_1 to W_n), connecting adjacent layers, are learned first via greedy layer-wise pretraining [20]. The weights can be fine-tuned in a supervised manner. However, this paper concentrates more on unsupervised pre-training for capturing the coherent structures in flame images at unstable state. Multi-variate time series of activation probabilities emanating from the hidden units of the bottleneck (last layer) hidden layer is fed into the STSA module as input. Synchronized pressure time series is also directed to STSA module for sensor fusion.

STSA learns D-Markov machines via symbolization and probabilistic finite state automata (PFSA) [13] generation from pressure data and individual time series arising from DBN bottleneck hidden units. It also models variable depth $\times D$-Markov machines which captures the cross-dependences among DBN hidden layer temporal output and pressure time series. This paper proposes to exploit this $\times D$-Markov machine construction for early detection of thermo-acoustic instability via spatiotemporal fusion. While the DBN efficiently reduces the dimension of the hi-speed video by preserving the semantic feature (continuing presence of coherent structures [14] in hidden layer visualization), STSA models temporal evolution of that feature via constructing $\times D$-Markov machine that considers heterogeneous sensors spatially dislocated. Elements of the proposed tool chain are explained in detail later in the sequel.

A. Deep Belief Network

Deep Belief Networks (DBN) is a type of deep neural networks consisting of multiple hidden layers of latent variables. They are constructed by stacking multiple layers of Restricted Boltzmann Machines (RBM) on top of each other, which are generative probabilistic graphical models with the capability to learn a probability distribution over the inputs to best explain the observed data. Individual RBMs are composed of visible units (the inputs) and hidden units that are interconnected but not among the same type of units. Due to the connections to latent variables, a single layer of RBM is powerful enough to represent complex distributions. The nonlinear modeling capacity is further increased when multiple hidden layers are stacked on top of each other, with the outputs of one becoming the input of another.

Multi-layered deep neural networks are notoriously difficult to train as the parameter optimization process can often get caught in poor local minima due to the large number of parameters in the model. However, it has been discovered that DBNs can be trained in an unsupervised manner to help initializing better weights as opposed to using randomized weights, therefore leading to a superior generalization performance.

Pretraining is performed in a greedy layer-wise manner. The weights and biases of the first RBM stack is updated iteratively based on an unsupervised training criterion. After a user-defined stopping condition (e.g. maximum number of iterations), the parameters from this layer is fixed and the outputs of the layer (a new representation for the raw input) becomes the input of another layer for pretraining in a similar fashion. Essentially, the objective is to find the hidden unit features that are more common in the training inputs than in the random inputs, such that the pretrained weights may help to guide the parameters of that later towards better regions in the parameter space.

Consider a single RBM stack with hidden units **h**. The probability of observing a sample **v** is

$$\mathrm{Pr}(\mathbf{v}) = \frac{e^{-F(\mathbf{v})}}{\int e^{-F(\mathbf{v})}}$$

where

$$\mathrm{F}(\mathbf{v}) = -\log \sum_{h} e^{-\mathrm{E}(\mathbf{v},\mathbf{h})}$$

and

$$\mathrm{E}(\mathbf{v},\mathbf{h}) = -\mathbf{b}^T\mathbf{v} - \mathbf{c}^T\mathbf{h} - \mathbf{h}\mathbf{W}^T\mathbf{v}$$

Pretraining seeks to find the set of parameters $\{\hat{\mathbf{W}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}\}$ (i.e., layer weights, visible unit biases and the hidden unit biases, respectively) that maximizes the expected log-likelihood of the training data **V**. Thus, the optimization problem can be formally represented as:

$$\{\hat{\mathbf{W}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}\} = \arg\max_{\mathbf{W},\mathbf{b},\mathbf{c}} \mathbb{E}\left[\sum_{\mathbf{v}\in\mathbf{V}} \log \mathrm{Pr}(\mathbf{v})\right]$$

and the problem is typically solved via stochastic gradient descent. In addition, each newly pretrained layer guarantees an increase on the lower-bound of the log-likelihood of the data, hence improving the model.

The whole pretrained network is finetuned using an error backpropagation algorithm. For a classification problem, the class labels are compared against the neural net outputs based on an input vector via an error metric that becomes the cost function of the algorithm. Specifically, the loss function ℓ to be minimized for a dataset **V**, parametrized by θ is:

$$\ell(\theta = \{\mathbf{W},\mathbf{b},\mathbf{c}\}, \mathbf{V}) = -\sum_{i=0}^{|\mathbf{V}|} \left[\log\left(\mathrm{Pr}(Y = y^{(i)}|v^{(i)}, \mathbf{W},\mathbf{b},\mathbf{c})\right)\right]$$

where $y^{(i)}$ denotes the class index. All weights and biases in the network are then optimized by the algorithm to produce a fully trained model that is capable of making class predictions based on a certain input.

B. Cross-Modeling of Symbolic Processes for Sensor Fusion

In this section, first we very briefly introduce some basic concepts of symbolic time series analysis followed by the $\times D$-Markov modeling framework for inferring the causal-cross dependence between two observed symbol sequences which we use for fusion. Interested readers are referred to [6], [13] for detailed discussion.

Symbolic time-series analysis (STSA) is a non-linear technique for modeling temporal patterns in sequential data. Symbolic analysis of time-series data for precise modeling of the underlying dynamics needs two challenges to be satisfied simultaneously.

1) *Symbolization*: The process of projecting continuous time-series data onto a symbolic domain.
2) *Depth Estimation:* The task of estimating the temporal memory of the system.

Once the data has been symbolized and the depth for the symbol sequence been estimated, the symbol stream can be compressed into generative models as Probabilistic Finite State Automata (PFSA). Next we present some relevant definitions to present things more clearly.

Definition III.1 *(D-Markov [13]) A D-Markov machine is a statistically stationary stochastic process $S = \cdots s_{-1}s_0s_1\cdots$ (modeled by a PFSA in which each state is represented by a finite history of D symbols), where the probability of occurrence of a new symbol depends only on the last D symbols, i.e.,*

$$P[s_n \mid \cdots s_{n-D}\cdots s_{n-1}] = P[s_n \mid s_{n-D}\cdots s_{n-1}] \quad (1)$$

where, D is called the depth of the Markov machine.

Definition III.2 *(Entropy Rate [6]) The entropy rate of a PFSA (Σ, Q, δ, π) is defined in terms of the conditional entropy as follows.*

$$\begin{aligned} H(\Sigma|Q) &\triangleq \sum_{q\in Q} P(q)H(\Sigma|q) \\ &= -\sum_{q\in Q}\sum_{\sigma\in\Sigma} P(q)P(\sigma|q)\log P(\sigma|q) \end{aligned} \quad (2)$$

where $P(q)$ is the probability of a PFSA state $q \in Q$.

In $\times D$-Markov machines, the Markov assumption is made on the expected outcome of a symbolic process conditioned on the states of another symbolic process (instead of states of the same symbolic process). In $\times D$-Markov machines, we assume that a symbol block of (finite) length D is sufficient to describe the current state for the PFSA constructed from the symbol stream $\{\mathbf{s}_1\}$. In other words, the symbols that occur prior to the last D symbols do not affect the subsequent symbols observed in symbol stream $\{\mathbf{s}_2\}$. We define a $\times D$-Markov machine next.

Definition III.3 ($\times D$-Markov) *Let K_1 and K_2 be the PFSAs corresponding to symbol streams $\{\mathbf{s}_1\}$ and $\{\mathbf{s}_2\}$, respectively. Then, an $\times D$-Markov machine (from $\{\mathbf{s}_1\}$ to $\{\mathbf{s}_2\}$) is* *defined as a 5-tuple* $\mathcal{M}_{1\to 2} \triangleq (\mathcal{Q}_1, \Sigma_1, \Sigma_2, \delta_1, \Pi_{12})$ *such that:*

1) $\mathcal{Q}_1 = \{q_1, q_2, \ldots, q_{|\mathcal{Q}_1|}\}$ *is the state set corresponding to symbol sequence* $\{\mathbf{s}_1\}$
2) $\Sigma_1 = \{\sigma_0^1, ..., \sigma_{|\Sigma_1|-1}^1\}$ *is the alphabet set of symbol sequence* $\{\mathbf{s}_1\}$
3) $\Sigma_2 = \{\sigma_0^2, ..., \sigma_{|\Sigma_2|-1}^2\}$ *is the alphabet set of symbol sequence* $\{\mathbf{s}_2\}$
4) $\delta_1 : \mathcal{Q}_1 \times \Sigma_1 \to \mathcal{Q}_1$ *is the state transition mapping for* $\mathcal{M}_1$
5) Π_{12} *is the cross morph matrix of size* $|\mathcal{Q}_1| \times |\Sigma_2|$*; the* ij^{th} *element* $(\pi_{12}(q_i, \sigma_j^2))$ *of* Π_{12} *denotes the probability of finding the symbol* σ_j^2 *in the symbol string* $\{\mathbf{s}_2\}$ *at next time step while making a transition from the state* q_i *of the PFSA constructed from the symbol sequence* $\{\mathbf{s}_1\}$*.*

Analogous to entropy-rate of a PFSA, we define a cross-entropy rate for a *crossed* PFSA which we will use as a metric for constructing the states of the cross-PFSA model for two observed symbolic processes.

Definition III.4 *($\times D$-Markov Entropy Rate [6]) The $\times D$-Markov entropy rate from a PFSA* $(\Sigma_1, Q_1, \delta_1, \pi_1)$ *to a symbol stream (say,* $\{\mathbf{s}_2\}$ *with alphabet set* Σ_2*) is defined as:*

$$H(\Sigma_2|Q_1) \triangleq \sum_{q_1\in Q_1} P(q_1)H(\Sigma_2|q_1)$$
$$= -\sum_{q_1\in Q_1}\sum_{\sigma_1^2\in\Sigma_2} P(q_1)P(\sigma_1^2|q_1)\log P(\sigma_1^2|q_1) \tag{3}$$

where $P(q_1)$ *is the probability of a PFSA state* $q_1 \in Q_1$ *and* $P(\sigma_1^2|q_1)$ *is the conditional probability of a symbol* $\sigma_1^2 \in \Sigma_2$ *given that a PFSA state* $q_1 \in Q_1$ *is observed.*

The number of states of a $\times D$-Markov machine of depth D is bounded above by $|\Sigma_1|^D$, where $|\Sigma_1|$ is the cardinality of the alphabet Σ_1. However, from the perspective of modeling the cross-dependance from $\{\mathbf{s}_1\}$ to $\{\mathbf{s}_2\}$, some states may be more important than others in terms of their embedded causal information contents. Thus it might be redundant to keep all states of different depths in the cross-PFSA model. Instead it might be advantageous to keep a set of states that correspond to symbol blocks of different lengths i.e., make a system where different states have different memory or depth D. This is accomplished by starting off with the simplest set of states (i.e., $Q_1 = \Sigma_1$ for $D = 1$) and subsequently splitting the current state that results in the largest decrease of the $\times D$-Markov entropy rate $H(\Sigma_2|Q_1)$ (see Eq. (3)). Thus we can restrict the exponential growth of states with increasing depth D without compromising the details of symbolic dynamics. At each step of state splitting, each element $\pi_{12}(\sigma_2, q_1)$ of the cross morph matrix Π_{12} is estimated numerically by frequency counting as the ratio of the number of times, $N(q_1\sigma_2)$, the state q_1 from $\{\mathbf{s}_1\}$ is followed by the symbol σ_2 from $\{\mathbf{s}_2\}$ and the number of times, $N(q_1)$, the state q_1 occurs. A stopping rule for state splitting is constructed by specifying the threshold parameter η_{spl} on the rate of decrease of $\times D$-Markov entropy rate.The final estimated morph matrix $\widehat{\Pi}_{12}$ can then used as representative feature of causality from the first to second symbolic process. This can then be used to perform various machine learning operations like pattern matching, classification, regression, clustering etc.

IV. Results and Discussion

This section discusses the results that are exhibited when the proposed framework is applied on the experimental data for early detection of thermo-acoustic instability via spatiotemporal fusion of hi-speed video and pressure data. For learning the DBN from video data, a network with three hidden layers of size 1000, 100 and 10 respectively is chosen while keeping the volume of training data under consideration. After scaling down by 4, the 56×98 pixels frame at the entry of the flame serves as the input vector (1×5488 after flattening) which is fed into the DBN at each time instant. Data from all four conditions at partial premixing ($X_1 = 90mm$) and two conditions at full premixing ($X_1 = 90mm$, $FFR = 0.495g/s, Re = 7,971; 15,942$) containing 54000 images in total are used for training the DBN via stochastic gradient descent method as explained in Subsection III-A. For both pre-training and supervised finetuning, learning rate of 0.01 is used for the gradient descent algorithm. This study mainly emphasizes on the last hidden layer output after pretraining because previous work by the authors [14] suggested that even pretrained hidden layers capture significant amount of coherent structures in the flame without supervised learning. Eventually, successful modeling of flame coherent structure is important during dimensionality reduction of images for constructing a scaler measure that detects instability early.

Once the DBN is learned via layer-wise pretraining, test video data containing three operating conditions at full premixing are fed into the network to obtain the last hidden layer representation by feed-forward operation. The details of the three conditions are as follows: (i) $FFR = 0.66g/s$, $Re = 10,628$ at stable state, (ii) $FFR = 0.083g/s$, $Re = 1,771$ at relatively stable state and (iii) $FFR = 0.308g/s$, $Re = 10,628$ at thermo-acoustically unstable state. During the testing phase, temporal sequence of high dimensional images (i.e., 5488 pixels) generates multi-variate time series with few hidden unit (= 10) activation probabilities at bottleneck hidden layer of the DBN. Thus, fusion of large dimensional hi-speed video and pressure time series is reduced to a problem of fusing the DBN bottleneck layer time series and pressure time series.

The STSA module constructs generalized D-Markov machine [21] from individual time series and models the cross-dependence from one time series to another by constructing $\times D$-Markov machine. $\times D$-Markov machines from bottleneck layer time series (for each hidden unit) to synchronized pressure time series are obtained by the process explained in Subsection III-B. The direction of $\times D$-Markov machine

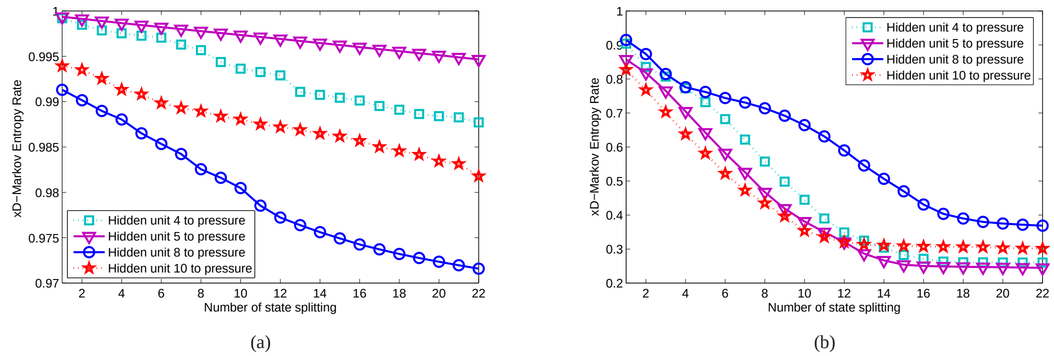


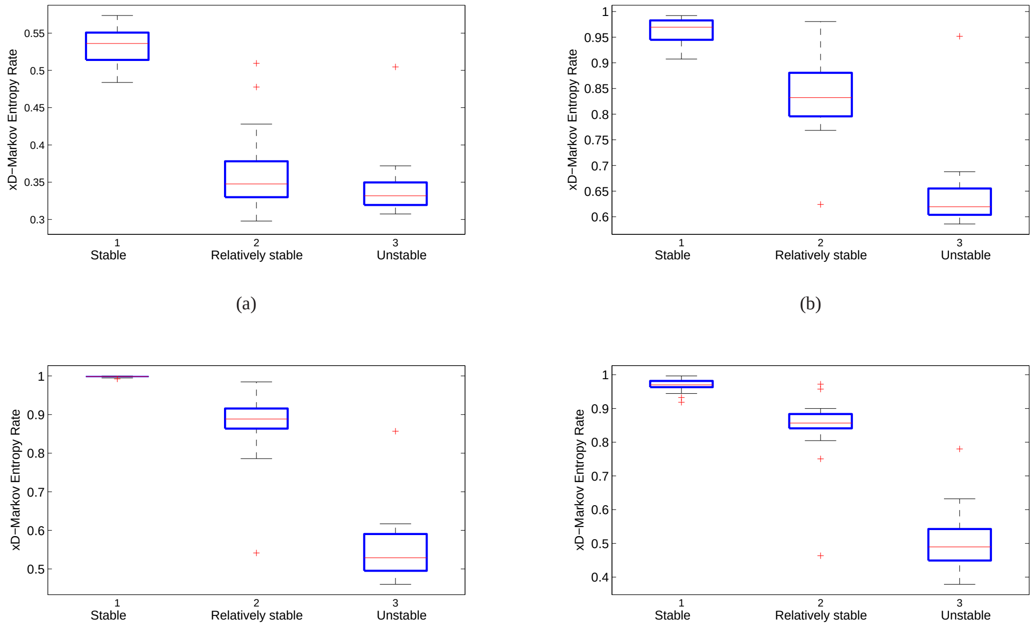
Fig. 2: Variation of $\times D$-Markov entropy rate directed from time-series of DBN hidden units (output of hi-speed video) to pressure time series as a function of number of state splitting at a symbol size $|\Sigma| = 3$ for (a)stable combustion, (b) thermo-acoustically unstable combustion

construction is identified to be from hi-speed video bottleneck layer to pressure data because pressure sensor is located downstream of the imaging sensor according the flow. The time series for both modalities are symbolized by maximum entropy partitioning (MEP) [13] with an alphabet size of $|\Sigma| = 3$. Figure 2 shows the drop of $\times D$-Markov entropy rates with increasing number of state splitting while constructing the $\times D$-Markov machines directed from bottleneck layer hidden units to pressure time series. For stable state, $\times D$-Markov entropy rate doesn't decrease significantly to a definite knee point as the time series itself is highly chaotic [7]. Based on η_{spl}, a cross-PFSA model can be selected in this scenario. Right half of Figure 2 shows that $\times D$-Markov entropy rate for 10th hidden unit to pressure time series converges earliest among others after 12 splits (i.e., 27 state $\times D$-Markov machine). Based on this analysis, the $\times D$-Markov machine directed from 10th hidden unit to pressure data can be chosen for further fusion operations.

When stable combustion becomes thermo-acoustically unstable, chaotic pressure time series becomes hi-amplitude periodic tones and phase differences among spatially apart sensors reduces drastically [18]. To capture this variation of temporal complexity and spatial dynamics among combustor sensors during the onset of instability, $\times D$-Markov entropy rate is proven to be a good candidate for constructing a sensitive and robust measure [12]. As instability creeps in, $\times D$-Markov entropy rate reduces because temporal complexity and phase difference deteriorate. Entropy rates are calculated on every 0.3 seconds window with 0.2 overlap based on the model learned via state splitting. There are 3 sec long temporal data coming from each bottleneck hidden unit and pressure time series for either of stable, relatively stable and unstable states. Computational complexity of the proposed approach is adequate to meet the requirement for combustion instability control, which is $10Hz$ response speed. Figure 3(a) presents a boxplot exhibiting a premature drop of pressure entropy rate along with a high variance over multiple time windows. Thus, entropy rate based on only pressure data may cause severe false alarm while detecting early onset of instability. This observation motivates the requirement of multi-modal fusion for this purpose. To compare with the proposed approach, Principal Component Analysis (PCA), a well-known dimensionality reduction technique is used as a replacement of DBN module. Aggregate time series of 10 largest variance components of test images are extracted based on PCA coefficients learned on same training images. It is observed in the boxplot of Figure 3(b) that $\times D$-Markov entropy rate directed from video generated PCA-aggregate time series to pressure time series is performing better than pressure entropy rate. However, it still has a high variance and low median at relatively stable state, which might cause false alarm in online prediction. Figures 3(c), (d) show the boxplots of $\times D$-Markov entropy rate directed to pressure time series from hi-speed video bottleneck layer 5th hidden unit and 10th hidden unit respectively. It is observed that the variance over time window is lower (especially for 10th hidden unit) compared to top half of the figure, making this measure more robust for real-time instability control. Overall, the negative slope of entropy rate drop for 'DBN+STSA' is higher than that of pressure or 'PCA+STSA' approaches. These observations denote that the $\times D$-Markov rate obtained via proposed 'DBN+STSA' fusion approach is a sensitive and robust measure for early detection instability.

V. Conclusions and Future Work

Combustion instability is undesirable and needs to be detected as early as possible for decision & control of gas turbine engines. This paper presents a dynamic data-driven approach for early detection of precursors to the instability phenomena using multimodal sensors and a combination of neural network and symbolic dynamic tools. Hi-speed images are first analyzed through deep belief networks (DBN) and then low-dimensional features are extracted in an unsupervised fashion. The temporal features obtained from the hi-speed images are combined with pressure time-series data by creating a variable-memory $\times D$-Markov model. The cross-entropy obtained from the $\times D$-Markov models of image with pressure data is used as the anomaly measure for detecting departure from stable operation in the combustion process. The proposed sensor-fusion approach has been validated on experimental data obtained from a laboratory-scale swirl-stabilized combustor apparatus. Usage of the sensor fusion algorithm for calibration of low-fidelity sensors by hi-fidelity sensors like video is recommended as a future work.



(a) (b) (c) (d)

Fig. 3: Entropy rate as an instability measure at stable, relatively stable and unstable states for (a) D-Markov machine on pressure time series, (b) $\times D$-Markov machine directed from hi-speed video dominant-PCA-feature-aggregate time series to pressure time series, (c) $\times D$-Markov machine directed from hi-speed video bottleneck layer 5th hidden unit to pressure time series and (d) 10th hidden unit to pressure time series

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