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Topology Control in Mobile Sensor Networks using Information Space Feedback

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***Abstract*— Time-varying network topology plays a key role in mobile sensor networks for detection of an event of interest and subsequent awareness propagation within a monitoring and surveillance framework. While physical space parameters such as communication range and mobility characteristics directly drive the network structure, feedback from the information space can be useful to improve network topology and facilitate efficient information management. In this context, the paper proposes a feedback control scheme for tuning key network topology parameters, such as average degree and degree distribution under the recently proposed generalized gossip framework for distributed belief/awareness propagation in mobile sensor networks. The crux of this decentralized control policy is to modify the timelines of the asynchronous belief update protocol depending on the node-level belief/awareness. Using a proximity network representation for a mobile sensor network, the paper presents both analytic and numerical results associated with topology control scheme as well as its impacts on belief/awareness propagation characteristics.**

1. Introduction and Motivation

Rapid development in wireless communication, micro-electromechanical systems (MEMS) and digital technology [1] enabled large-scale implementation of wireless sensor networks (WSN) into various applications such as process monitoring, military reconnaissance, tactical surveillance, safety control, and resource operations. A network of sensors can be conveniently deployed in air, underwater, and inside buildings for threat detection and threat tracking (e.g. vehicles, personnel, chemical and biological agents) [2] over a large region. Hence, the dynamics and sensor interactions need to scale well with the physical coverage area.

Currently, much work has been reported on consensus-based source-seeking methods [3] [4] to identify the source of events of an unknown signal field. Authors in [5] [6] [7] present a multi-agent coordination framework where the mobile agents collectively estimate the peaks of sensor field and move to the peak according to the estimated gradient. In [6], the agents need to maintain a formation for accurate gradient estimation. Authors in [8] [9] [10] [11] present algorithms for estimating the source location by using a stochastic gradient descent algorithm based on robot dynamics.

Multi-agent consensus problems are traditionally modeled as a discrete time system with states updated *synchronously* in a single pass before the new states influence the states of the other agents. However, many real-world systems are inherently *asynchronous*. The present work uses a gossip broadcasting algorithm proposed by Sarkar et al. [12] for information consensus (modeled as a wireless proximity network [13]) in an asynchronous manner where each agent performs state updates at different time instants due to the varying lengths of update intervals.

Numerous works aim to improve the rate of information dissemination in wireless networks by changing network topology. Some authors rewired nodes to a random destination node [14] [15]. In the virus model, special agents control residual information using control laws involving binary state variables [16]. Predictive control [17] [18] are also used, although the predictive component may increase overhead of energy consumption. Another author established a hierarchy in agent responsibilities [19].

Frequently, improving topological lifetime of the network physically requires a larger financial investment in additional infrastructures. Hierarchical networks demand using a variety of sensors; more sensors are needed to increase sensor density in a region. Laying sensors in an optimal formation is not conveniently transferable to a different setting, whereas increasing the mobility of the agents results in higher energy consumption. We propose a method to improve network topology without altering the physical aspects of the network.

In consensus scenarios, a convergence set point can be defined and the behavior of the system can be analyzed rather easily. Oftentimes, the state transition matrix is required to be doubly stochastic (i.e., the rows and columns all sum to one) in order to prove convergence conveniently [20] [21] [22]. However, the set point in this problem depends on the closed-loop proportional controller gain and the update interval which is time-varying. Hence, the complexity increases because the doubly stochastic properties of the matrix is now absent as a consequence of the system being *linearly time-varying*. We show that the expectation in time of the state transition matrix is in fact doubly stochastic and helps proving convergence properties.

Contributions: The crux of the idea is to change the topological properties of the mobile sensor network by using only the feedback of a node's belief measure to reactively modify its update interval. This technique can be applied to spatially change network topology to increase the rate of convergence by minimizing the second-largest eigenvalue of the transition matrix without additional energy consumption. Furthermore, the framework uses continuous state variable for gossip as opposed to binary variables which is potentially applicable to a larger variety of problems and focuses on a single type of mobile sensor instead of static sensors assigned with different roles or to hierarchy of some kind.

2. Background on Generalized Gossip Protocol in Proximity Networks

This section provides a basic overview of proximity networks (also referred to as moving neighborhood networks [23]) and the generalized gossip algorithm. In practical applications, long distance communications are prone to issues such as delayed transmission and data degradation. These issues are detrimental to systems undertaking time-critical missions in hostile environments. For this reason, distributed proximity networks are essential when technical limitation restricts the ability of agents to transmit information to a central information sink (e.g., GPS satellite) and prohibits the agents from executing intelligent decisions. As a matter of clarity, the terms *mobile sensor*, *node* and *agent* are used interchangeably.

In the original problem formulation [12], each mobile sensor establishes a link with proximal agents within a predefined communication range R within a fixed update interval. This update interval is defined below as *message lifetime* L.

Definition 2.1 *(Message lifetime, L) Agent belief update interval. When L for an agent expires, the agent performs an update of its own belief while accounting for the belief of other agents within the neighborhood of the agent. Subsequently, all established information links originating from that particular agent, along with its local message lifetime, is reset and the process is repeated. As $L \to 0$, the agent could never form links with other agents fast enough. On the other hand, when $L \to \infty$, the network of agents will eventually become fully connected.*

Remark 2.1 *The physical dynamics of the agents (e.g., agent motion, and the ticking of clock that keeps track of message lifetime L) progress in real time, referred to as the fast time scale. The information space (i.e., network topology) evolves on a slower time scale where each slow time instant τ refers to the moment when agents update their belief upon the expiration of message lifetime L.*

Interaction between agents and the flow of information is dictated by the *decentralized generalized gossip algorithm* which has the following expression $\nu_\theta^{(i)}|_{\tau+1} = (1 - \theta)\sum_{j\in i\cup N_{b(i)}} \Pi_{ij}|_\tau \nu_\theta^{(j)}|_\tau + \theta\chi^{(i)}|_\tau$, or more simply, in vector form:

$$\nu_\theta|_{\tau+1} = (1-\theta)\Pi|_\tau \nu_\theta|_\tau + \theta\chi|_\tau \tag{1}$$

where $\nu_\theta \in [0, 1]$ denotes the agent measure, or *belief*, of a node indicating the level of awareness (0 for no awareness, 1 for complete awareness) regarding the presence of a localized hotspot in the environment. The state characteristic function, or *observation* $\chi \in \{0, 1\}$ is a binary variable which describes whether the agent has actually detected the target ($\chi = 1$) or not ($\chi = 0$). The parameter θ is another important parameter which has the following definition:

Definition 2.2 *(Control parameter, θ) Parameter to control emphasis on either agent belief or the agent state characteristic function, where $\theta \in (0, 1]$. As $\theta \to 1$, the sensors rely more on its own observation rather than its own belief combined with the beliefs of neighboring sensors. As $\theta \to 0$, the agent updates its belief based solely on other agents in the neighborhood without considering whether or not it has actually detected the hotspot.*

Note that the subscript θ in ν_θ indicates that the agent belief evolution is parametrized by the control parameter θ. $N_{b(i)}$ in (1) denotes the set of sensors communicating with the i-th sensor during the communication window between slow-time instant τ and the next instant $\tau + 1$.

The Π term denotes the interaction matrix describing the connectivity among agents. To compute Π, the graph Laplacian matrix $\mathcal{L}$ is required and is defined as $\mathcal{L} = D - A$ where the i-th diagonal element in degree matrix D corresponds to the degree of the node i. The adjacency matrix A is defined such that the element in the ij-th and ji-th position is unity if two agents i and j have established a link before L expires. Otherwise, the value is zero. Matrix A is also defined where the element $a^{ij} = 0$ for $i = j$. Now, the interaction matrix Π is $\Pi = \mathrm{I} - \beta\mathcal{L}$ where I is the identity matrix and $\beta = 1/(\bar{d} + 1)$. In the setup, the agent ignores communications with other agents that are beyond $\bar{d}$ agents within message lifetime L. The value of $\bar{d}$ is chosen such that the probability of degree $d^{(i)} > \bar{d}$ for agent i is less than 0.001 (for the study). Π becomes a stochastic matrix when the product $\beta\mathcal{L}$ is subtracted from the identity matrix I. Further details can be found in [12].

3. Problem Formulation and Asynchronous Belief Updates

Sensor network as proximity network: Consider a network with multiple mobile sensors deployed to detect threats in a region (modeled as a hotspot with a given radius). Sensors move around in a random walk fashion and have a nonzero probability of detecting threats upon entering the hotspot. While threat detection does not influence the mobility of the sensors, all sensors exchange information of what they know from the environment at specific intervals.

Sensor states are updated to reflect the level of awareness towards the threat. The goal is to disseminate information away from the hotspot to the entire sensor population.

Synchronous and asynchronous updates: In the original formulation with homogeneous message lifetime L presented in [12], all connections are bidirectional at all times; the links between two connected sensors i and j are mutual. Since the beliefs of all sensors update synchronously due to homogeneous L, the Π matrix is stochastic and symmetric, i.e., doubly stochastic. However, synchronous update requires the internal clock of all sensors to be in sync, which is not achieved easily in reality. For example, sensors responsible for undersea surveillance operations have low power and low fidelity to minimize the frequency of replacing the battery supplies. Routine maintenance is inevitable; if the internal clocks are not recalibrated upon redeployment, sensors would update their beliefs asynchronously.

Similarly, an update rule that modifies a sensor's update interval based on its own belief measure results in asynchronous updates. The state transition matrix Π is no longer symmetrical and doubly stochastic at every slow time instant. With sufficient randomly-walking mobile sensors, the time-averaged expected value of Π matrix can still be approximated as a doubly stochastic matrix and numerical simulations presented later in this paper confirms this conjecture.

The experimental parameters in the setup consist of a 2-dimensional operational region of length $l = 100$ units, area $A = l^2 = 10^4$ units and hotspot length scale $\lambda = r_h/l = 0.1$, where r_h is the radius of the hotspot. Agent density is defined to be $\rho = N/A = 0.01$ and N is the total number of agents in the network. The control parameter is fixed at $\theta = 0.01$ and base message lifetime $L_b = 15$. All sensors move with constant velocity $v = 10$ units in a random walk fashion.

4. Network Topology Control with Message Lifetime Actuation

This work proposes a modification to the update rule from the generalized gossip algorithm to enable dynamic topological evolution for quicker information propagation in an asynchronous manner. For this problem, various factors affecting network connectivity can be considered, such as the communication radius, agent density in the operational field, agent velocity, and the duration of message lifetime. Clearly, increasing these parameters result in the ability of forming more connections over a set period of time, but some may not be necessarily feasible in practical applications.

Recall in section 2 that when $L \to \infty$, the network becomes fully connected. Therefore, setting L as a constant with high value is undesirable because there would not be a localized gradient of belief given a long enough time. Hence, the paper proposes an algorithm to control information propagation by setting the message lifetime $L^{(i)}$ of a sensor i as a function of its belief $\nu^{(i)}$ with the following expression:

$$L^{(i)} = L_b(1 + P\nu^{(i)}) \tag{2}$$

where $L^{(i)}$ is the message lifetime for agent i, L_b is the base message lifetime predetermined off-line and P can be thought as the proportional gain of the closed-loop system.

Definition 4.1 *(Proportional Gain for Message Lifetime Control, P) A tuning parameter that proportionally influences the new duration of message lifetime L in the next slow time instant after the agent performs an update.*

Remark 4.1 *Varying L does not negatively impact the overall power consumption as it does not require altering hardware behaviors (e.g., increasing motor speed to move faster) in a sensor. L at all time instants will only be larger or equal to the user-predetermined base message lifetime L_b so there will be no additional power consumption due to increased update frequency.*

For stochastic analysis performed here, it is important to distinguish between *ensemble expectation* $\mathrm{E}_e[\mathbf{x}] = \frac{1}{N}\sum_{k=1}^{N} x_k$, that is, the expected value averaged over all sensors at a time instant; and the *time-averaged expectation* $\mathrm{E}_t[x] = \lim_{T\to\infty} \frac{1}{T}\sum_{k=0}^{T} x(k)$. Recall that message lifetime L of the i-th sensor updates according to the rule described in (2), which results in nonhomogenous L and hence asynchronous belief updates. Taking the ensemble expectation on both sides of (2), the expression becomes:

$$\mathrm{E}_e[L^{(i)}] = L_b + L_b P \mathrm{E}_e[\nu^{(i)}] \tag{3}$$

This establishes the basic linear relationship between the ensemble expectation of the message lifetime L and the ensemble expectation of agent measure ν. The topology of the network can be statistically represented by the degree distribution of the network. Under the current problem formulation, degree (denoted as k) of a sensor is defined to be the number of outbound connections with other agents in the network, where $\Pr(k)$ for the network is defined to be the probability distribution of node degrees over the network. Let $\Pr(k|L,i)$ be the distribution of the number of distinct nodes that communicate with a given node i within message life L. The degree distribution can be written as $\Pr(k|\bar{L}) \triangleq \frac{1}{n(\bar{L})}\sum_{i:L(i)=\bar{L}} \Pr(k|\bar{L},i)$, where $n(\bar{L})$ is the number of nodes with message lifetime L. Then, the overall degree distribution of the network is obtained by taking the expected value of $\Pr(k|L)$ to get $\Pr(k) \triangleq \frac{1}{N}\sum_{L} n(L)\Pr(k|L)$.

For ensemble analysis, let the message lifetime for node i (i.e., $L^{(i)}$) be taken as $\mathrm{E}_e[L]$ for all nodes. Assuming independent activities at each fast time instant, the probability of two distinct nodes i and j not communicating with each other within the expected message lifetime $\mathrm{E}_e[L]$ is $1 - (\alpha g_i g_j)^{\mathrm{E}_e[L]}$, where α is a parameter describing the spatial information of the network (e.g., sensor density). It

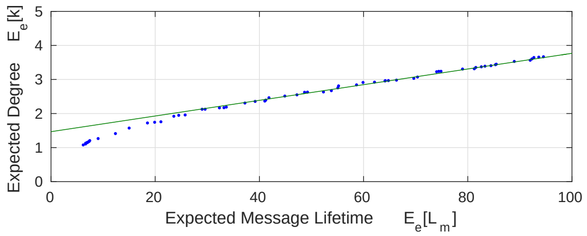


Fig. 1. Expected degree $\mathrm{E}_e[k]$ vs. expected message lifetime $\mathrm{E}_e[L]$: The relationship between these two variables are approximately linear when

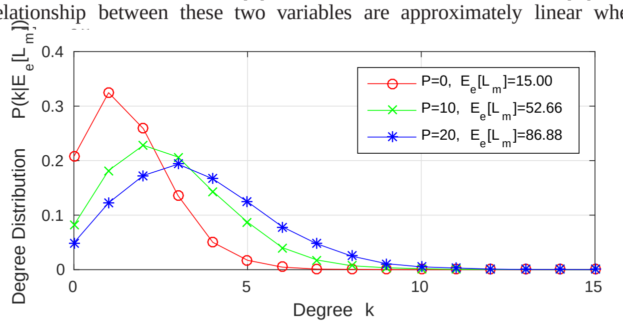


Fig. 2. Plot of degree distribution $\Pr(k|\mathrm{E}_e[L])$ of the network for (1) $P = 0, \mathrm{E}_e[L] = 15$; (2) $P = 10, \mathrm{E}_e[L] = 52.66$; (3) $P = 20, \mathrm{E}_e[L] = 86.88$. Degree distribution is found to be Poisson in nature.

follows that $p_{ij}(\mathrm{E}_e[L]) = 1 - (1 - \alpha g_i g_j)^{\mathrm{E}_e[L]}$. Terms g_i and g_j refers to the *gregariousness* or the tendency of an agent to communicate with another [13]. For all agents, g_i and g_j are constant because both agent velocity and the radius of communication are time invariant. It has been shown in [12] that the degree k of node i has the following relationship with α, g_i, and g_j where $k_i = \sum_{j=1}^{N} p_{ij}(L) \simeq L(\alpha g_i \sum_j g_j)$ for $\alpha L \ll 1$. Hence, with nonhomogeneous message lifetime under the current formulation and taking the ensemble expectations, the equation becomes:

$$\mathrm{E}_e[k] \simeq \mathrm{E}_e[L](\alpha g_i \sum_j g_j) \text{ for } \alpha \mathrm{E}_e[L] \ll 1 \quad (4)$$

Therefore, the ensemble expectation of degree $\mathrm{E}_e[k]$ is a linear function of $\mathrm{E}_e[L]$. This relationship is validated by numerical experiments and presented in Fig. 1.

With agents i and j having the same specifications, $p_{ij}(\mathrm{E}_e[L])$ is written simply as $p(\mathrm{E}_e[L])$. The degree distribution $\Pr(k|\mathrm{E}_e[L])$ can now be written as:

$$\begin{aligned}\Pr(k|\mathrm{E}_e[L]) &= \binom{N}{k} p^k (1-p)^{N-k} \\ &\simeq \frac{(\mathrm{E}_e[k])^k}{k!} e^{-\mathrm{E}_e[k]} \text{ for } N \gg 1 \end{aligned} \quad (5)$$

which takes the form of the equation for a Poisson distribution. Fig. 2 from numerical simulation shows that the degree distribution $\Pr(k|\mathrm{E}_e[L])$ for $P = 0, 10, 20$ (which yields corresponding $\mathrm{E}_e[L]$) is indeed Poisson. Note, in [12], Sarkar et al. have validated that the degree distribution is Poisson in nature in the synchronous case. The simulation in this paper, on the other hand, confirms the same outcome for the asynchronous case with proportional feedback control policy.

5. Convergence of Statistical Moments

Convergence of measure average: Due to the stochastic nature of the problem, procedures to prove statistical convergence involve instances of taking expected quantities. Although χ and ν fluctuate in the slow time scale at steady-state, the expected average $\mathrm{E}_t[\mathcal{M}_a[\cdot]]$ and variance $\mathrm{E}_t[\mathcal{V}_a[\cdot]]$ of ν over agents are considered to be at steady-state. Defining $\mathbf{x}$ as a column vector $\mathbf{x} = [x_1, x_2, ..., x_N]^T$, the average can be defined as $\mathcal{M}_a(\mathbf{x}) = \frac{1}{N}\mathbf{1}\mathbf{x} = \mathbf{x}^{avg}$ where $\mathbf{1}$ is a row vector of ones. In addition, with $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}^{avg}\mathbf{1}^T$ and superscript T denoting the transpose of a matrix, the variance over agents can be expressed as $\mathcal{V}_a(\mathbf{x}) = \tilde{\mathbf{x}}^T\tilde{\mathbf{x}}$. Since $\nu|_\tau$ is an $N \times 1$ vector, both average and variance values are scalar.

To show the convergence nature of the expected average measure ν over agents with respect to the expected average observation χ over agents at steady-state, we take the expectation of (1) in time to obtain:

$$\mathrm{E}_t[\nu_{\theta,P}|_{\tau+1}] = (1-\theta)E_t[\mathit{\Pi}|_\tau \nu_{\theta,P}|_\tau] + \theta \mathrm{E}_t[\chi|_\tau] \quad (6)$$

Since agent measure $\nu|_\tau$ at a slow time instant τ is not affected by the interaction matrix $\mathit{\Pi}|_\tau$, $\nu|_\tau$ and $\mathit{\Pi}|_\tau$ can be assumed to be independent. Hence, the equation can be rewritten as:

$$\mathrm{E}_t[\nu_{\theta,P}|_{\tau+1}] = (1-\theta)\mathrm{E}_t[\mathit{\Pi}|_\tau]\mathrm{E}_t[\nu_{\theta,P}|_\tau] + \theta \mathrm{E}_t[\chi|_\tau] \quad (7)$$

Note, the expectation of $\mathit{\Pi}|_\tau$ in time is doubly stochastic as discussed earlier. Therefore, after pre-multiplying both sides by $\frac{1}{N}\mathbf{1}$, the product involving $\mathit{\Pi}$ becomes $\mathbf{1}$ and the following expression is obtained:

$$\mathrm{E}_t[\nu_{\theta,P}^{avg}|_{\tau+1}] = (1-\theta)\mathrm{E}_t[\nu_{\theta,P}^{avg}|_\tau] + \theta \mathrm{E}_t[\chi^{avg}|_\tau] \quad (8)$$

Expanding the equation, one obtains:

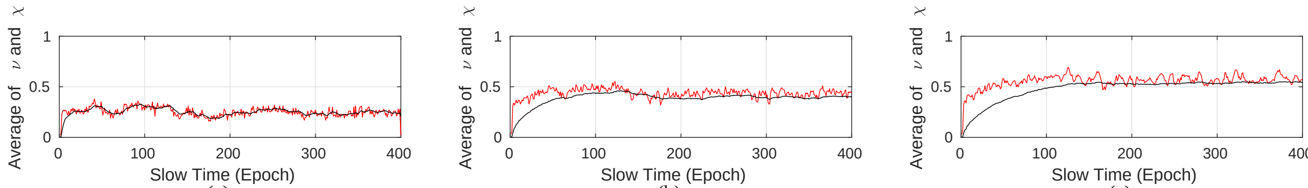
$$\begin{aligned}\mathrm{E}_t[\nu_{\theta,P}^{avg}|_{\tau+1}] = &(1-\theta)^{\tau+1}\mathrm{E}_t[\nu_{\theta,P}^{avg}|_0] + \theta \mathrm{E}_t[\chi^{avg}|_\tau] \\ &+ \theta(1-\theta)\mathrm{E}_t[\nu_{\theta,P}^{avg}|_{\tau-1}] \\ &+ \theta(1-\theta)^2\mathrm{E}_t[\chi^{avg}|_{\tau-2}] \\ &+ ... + \theta(1-\theta)^\tau \mathrm{E}_t[\chi^{avg}|_0] \end{aligned} \quad (9)$$

As $\tau \to \infty$, the equation simplifies to:

$$\begin{aligned}\mathrm{E}_t[\nu_\theta^{avg}|_\infty] &= \theta[1 + (1-\theta) + (1-\theta)^2 + ...]\mathrm{E}_t[\chi^{avg}] \\ &= \theta[1 - (1-\theta)]^{-1}E_t[\chi^{avg}] \\ &= \mathrm{E}_t[\chi^{avg}] \text{ for } \theta \in (0,1] \end{aligned} \quad (10)$$

that ultimately yields $\mathrm{E}_t[\mathcal{M}_a(\nu_{\theta,P})] = \mathrm{E}_t[\mathcal{M}_a(\chi)]$. This equality (when $\tau \to \infty$) shows that at steady-state, the sum of ν over agents is equal to the sum of χ over agents. The convergence in values implies that the decision of agents detecting the hotspot is being redistributed as awareness among other agents and signifies the conservation of total awareness measure in the system. Fig. 3 shows the plots obtained from simulation that validates the above relationship.

Furthermore, Fig. 4 is a dual-axis plot that shows the variation of average degree $\mathrm{E}_e[k]$ and average belief ν over



(a) (b) (c)

Fig. 3. Plots of average agent measure ν and observation χ over time with $P = 0$, 10 and 20 for panels (a), (b), and (c) respectively, at $\theta = 0.1$. The red curve denotes χ and black curve denotes ν. In all cases, $\mathrm{E}_t[\mathcal{M}_a(\nu_\theta)]$ converges to $\mathrm{E}_t[\mathcal{M}_a(\chi)]$. Note, θ is selected to be 0.1 instead of 0.01 during generation of the plot to clearly illustrate convergence between agent measure averages within 400 epochs. The convergence behavior in this case

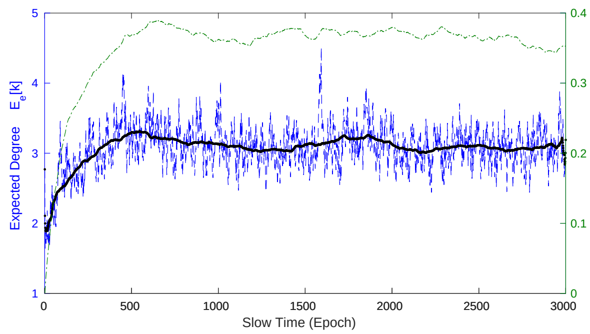


Fig. 4. Expected degree $\mathrm{E}_e[k]$ vs time for $\theta = 0.01$. Green dashed line at the top is the average belief, blue dashed line in the middle is the expected degree, and the solid black line through the middle is the 300-period moving average of the expected degree. Note that the fluctuations in the moving average degree correspond to the fluctuation in average belief (i.e., measure) over all agents.

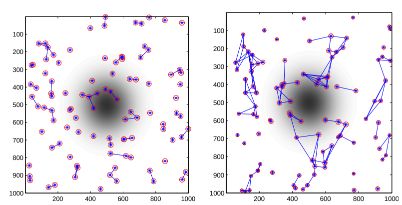

(a) (b)

Fig. 5. Simulation to demonstrate effects of P on agent degree. (a) shows the case where $P = 0$ and (b) shows the case where $P = 20$. Each dot represents an agent in the field with connecting lines indicating established connections. Larger red circles surrounding the agent represents a relatively longer message lifetime compared to other agents on a local basis. Note that the larger circle size in (a) does not indicate that the message lifetime is longer than in (b); comparison is performed locally.

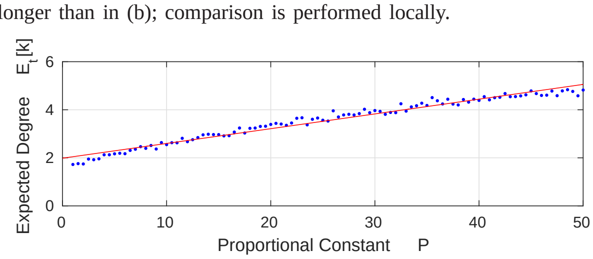


Fig. 6. Plot of expected degree $\mathrm{E}_e[k]$ vs proportional gain P. The plot shows an approximately linear relationship between $\mathrm{E}_e[k]$ and P.

time. Since the hotspot is active for the whole duration of this simulation, there would still be a fluctuation in ν and χ upon reaching steady state. However, the trend becomes clear when a 300-period moving average is taken. As observed, the belief ν converges to steady state and so does the degree $\mathrm{E}_e[k]$. This is expected because L is a function of ν.

Effects of proportional gain on the expected degree: Before, the convergence of measure variance is investigated. This section discusses the impacts of varying proportional gain P on the topology of the network that is represented by the degree of agents. As shown in Fig. 3, increasing P raises both average ν and χ. See Fig. 5 for an illustration of impact of P on the network topology. To understand the overall relationship, Fig. 6 from simulation shows how the ensemble expectation of node degree $\mathrm{E}_e[k]$ varies with different values of proportional gain P. As shown earlier, the expected degree varies linearly with expected message lifetime (see Eqn. (4)) and the expected message lifetime in turn follows a linear relationship with the proportional gain P (see Eqn. (3)). Therefore, the relationship between $\mathrm{E}_e[k]$ with P is also approximately linear as expected.

Convergence of measure variance: Analytical results for variance analysis consider two special scenarios under the congruous time scale (CTS) and the disparate time scale (DTS) [12]. In CTS, the time scales for both physical and informational dynamics remain comparable. In this scenario, at every slow time τ there exists an independent agent interaction matrix $\mathit{\Pi}$ and an independent state characteristic vector χ. To physically realize this scenario, agents must either have a sufficiently long message life L or move fast enough to decay the temporal correlations between the two slow time epochs. In CTS, the upper and lower bounds of measure variance are identified in [24] to be $\theta^2 \leq \frac{\mathrm{E}_t[\mathcal{V}_a[\nu_{\theta,P}]]}{\mathrm{E}_t[\mathcal{V}_a[\chi]]} \leq \frac{\theta^2}{1-(1-\theta)^2\Lambda_2}$,where Λ_2 is the second largest eigenvalue of $\mathrm{E}[(\mathit{\Pi}|_\tau)^T(\mathit{\Pi}|_\tau)]$. On the other hand, the two time scales in the DTS scenario are different enough such that the network evolution and agent state updates can be considered independently. Agent measures converge before there is a change in $\mathit{\Pi}$ and χ. The bounds in the DTS can be expressed by $\theta^2 \leq \frac{\mathcal{V}_a[\nu_{\theta,P}]}{\mathcal{V}_a[\chi]} \leq \frac{\theta^2}{[1-(1-\theta)\lambda_2]^2}$,where λ_2 is the second largest eigenvalue of $\mathit{\Pi}$.

With the introduction of proportional gain P in either case, it is discussed earlier that increase in the value of P will cause the expected degree to rise. The rise in degree widens the spectral gap (i.e., the difference between the largest and second-largest eigenvalues) and reduces the second-largest eigenvalue. Based on the upper bounds provided above for both CTS and DTS cases, an increase in P will therefore lower the upper bound of the variance ratio. Although lowering of upper bound does not ensure lowering of measure variance, numerical experiments show that it seems to be

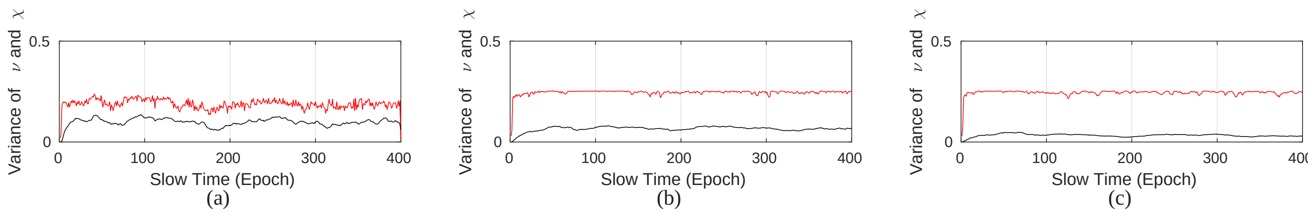


Fig. 7. Plots of agent measure ν and observation χ variances over time with $P = 0$, 10 and 20 for panels (a), (b), and (c) respectively, at $\theta = 0.1$. The red curve denotes χ and the black curve denotes ν. As P increases, $\frac{\mathrm{E}_t[\mathcal{V}_a[\nu_{\theta,P}]]}{\mathrm{E}_t[\mathcal{V}_a[\chi]}$ decreases. Note, θ is selected to be 0.1 instead of 0.01 for clarity. With $\theta = 0.1$, the magnitudes of variance are more easily compared by visual inspection. A similar trend is also observed in the original case where $\theta = 0.01$.

the case. Fig. 7 demonstrates the variation of the measure variance over time for different values of P.

6. Conclusion and Future Work

We used the feedback of a sensor node's belief measure to modify its own belief update interval which changes the network topology without physical modifications. Network topology is controlled by introducing a proportional gain P into the update rule relating message lifetime L as a function of agent measure ν. The network degree has a Poisson distribution and its expectation is a linear function of message lifetime L under this update scheme. Statistical moments and the expected network degree are shown to converge without causing the network to be fully connected. Analytical and simulation results show that varying the proportional gain P has a direct impact on network topology. Contrary to a constant L, a proportional feedback control policy involving gain P can easily control exploration versus exploitation in a highly dynamic manner. The following are future research directions:

1) Analysis of the relationship between proportional gain P and the rate of degree/belief convergence;
2) Analytical evaluation of expected characteristics of interaction matrix Π (e.g. the second-largest eigenvalues) with proportional gain P.

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