Building HVAC Systems Control Using Power Shaping Approach

V. Chinde, K. C. Kosaraju, Atul Kelkar, R. Pasumarthy, S. Sarkar, N.M. Singh

***Abstract*— Heating, Ventilating and Air-conditioning (HVAC) control systems play an important role in regulating indoor air temperature to provide building occupants a comfortable environment. Design of HVAC control system to provide an optimal balance between comfort and energy usage is a challenging problem. This paper presents a framework for control of building HVAC systems using a methodology based on power shaping paradigm that exploits passivity theory. The controller design uses Brayton-Moser formulation for the system dynamics wherein the mixed potential function is the power function and the power shaping technique is used to synthesize the controller by assigning a desired power function to the closed loop dynamics so as to make the equilibrium point asymptotically stable. The methodology is demonstrated using two example HVAC subsystems - a two-zone building system and a heat exchanger system.**

I. INTRODUCTION

Depleting natural energy resources and increasing costs are forcing all countries to look for technologies that can improve energy efficiency and not just generation of energy. It has been well documented that the costs of improving efficiency are much lower than the cost of generating equal amount of energy. Nearly 40% [1] of the total energy consumption in US is due to commercial and residential buildings. Heating, ventilation and air-conditioning (HVAC) systems are a major source of energy consumption in buildings. Statistics reveal that around 40% [2] of the energy used in commercial buildings is by HVAC systems. This makes it necessary to tackle energy related issues, such as thermal storage, in building systems by proper dynamic analysis and control design. Energy costs can be reduced by proper control of buildings thermal storage [3]–[5] and operating the buildings based on demand response [6]. These control techniques require accurate models which captures the thermal dynamics of the building. The models obtained should be such that they are computationally efficient so as to provide real time feedback inputs for control purposes, with conflicting objectives of energy efficiency and user comfort. The models presented in literature based on finite element methods for heat transfer dynamics in buildings have proven to be computationally inefficient [7]. Other prevailing modeling technique is Model Predictive Control (MPC) [8], [9]. In most cases, the zone temperatures are controlled using local controllers to ensure comfort of the occupants which typically leads to high energy consumption due to disparate energy demands from individual zones. One of the ways to capture the complex interconnection between mutiple zones, is to approximate the heat transfer model using an electrical (RC) network analogy [10]. Various zone modeling approaches have been recently compared in [11]. Once the complex dynamics is represented as an electrical network, one can use various tools from network theory (e.g., passivity-based methods) to devise interesting and novel control approaches [12].

Passivity [13] is an input-output property of physical systems that can be used for analysis and synthesis for complex systems. The underlying idea is to render a closed-loop system passive, by an appropriate feedback and assigning a desired closed loop storage (Lyapunov) function. In the context of (port-) Hamiltonian systems [14], this control technique is referred to as “energy shaping” where the objective is to shape the energy (the Hamiltonian) of the open-loop system. Another approach is the notion of power shaping, having its roots in the Brayton-Moser (BM) framework [15] for modeling of topologically complete nonlinear electrical networks with sources [16]. Passivity is derived using a power like function, also called the mixed potential function, as the storage function and one of the port variables being the derivative of voltages or currents. In this framework we describe the dynamics in terms of physical (or measurable) variables, such as voltages and currents in case of electrical networks. Moreover, since the derivatives of currents and voltages are used as measured outputs, it helps to speed up the transient response of the system. Finally, it overcomes the “dissipation obstacle” [17] encountered in classical energy shaping methods. The methodology can be used to solve the regulation problem in both finite [16] and infinite dimensional systems [18]–[20].

In this paper, we use the power shaping paradigm to design controllers for two different HVAC subsystems, namely thermal zones and heat exchangers. These representative examples were chosen as they demonstrate most of the typical complexities found in building HVAC systems. Although the models used are simple, its a good starting point and provide analysis as proof-of-concept and can be easily extended to include detailed building modeling which can serve different tasks. First, the dynamics of these two systems is transformed into the BM framework, then the input-output pair is identified that satisfies the passivity property. The control objective is then to assign a suitable power function to the closed loop system so as to make the equilibrium point asymptotically stable.

The organization of the paper is as follows. In Section II, we discuss power-shaping paradigm given the system dynamics in the BM form. In Section III, we give BM

V. Chinde, Atul Kelkar and S. Sarkar are with the Department of Mechanical Engineering at Iowa State University. K. C. Kosaraju, R.Pasumarthy are with the Department of Electrical and Computer Engineering at IIT Madras. N.M.Singh is with the Electrical Department at Veermata Jijabai Technological Institute (VJTI), India. Email: vchinde@iastate.edu

formulation for a multi-zone building model, and solve the temperature regulation problem using power shaping approach. The heat exchanger example is presented in Section III-C followed by conclusions presented in Section IV.

II. Power Shaping approach

This section briefly describes the underlying idea of power shaping.

A. Brayton-Moser form

In power shaping the dynamics of the system are written in gradient form using Brayton-Moser formulation, where the storage function has units of power. The gradient structure in the system is exploited to achieve power shaping outputs. Consider the standard representation of a system in Brayton-Moser formulation

$$Q(x)\dot{x} = \nabla_x P(x) + G(x)u \tag{1}$$

the system state vector $x \in \mathbb{R}^n$ and the input vector $u \in \mathbb{R}^m$ $(m \leq n)$. $P : \mathbb{R}^n \to \mathbb{R}$ is a scalar function of the state, which has the units of power also referred to as mixed potential function since in electrical networks it is the combination of content and co-content functions and the power transfer between the capacitor and inductor sub systems [21], $Q(x) : \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^n$ and $G(x) : \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^m$. The time derivative of the mixed potential functional is

$$\begin{aligned} \frac{d}{dt}P(x) &= \nabla_x P(x) \cdot \dot{x} \\ &= (Q(x)\dot{x} - G(x)u) \cdot \dot{x} \\ &= \dot{x}^\top Q(x)\dot{x} - u^\top G(x)^\top \dot{x} \end{aligned}$$

This suggests that if $P(x) \geq 0$ and $Q(x) \leq 0$, the system (1) is passive with storage function $P(x)$ and port power variables are input u, output $y = -G(x)^\top \dot{x}$. But, in general $P(x)$ and $Q(x)$ can be indefinite [16].

Assumption:

1) For the given system, there exists $\tilde{P}(x) \geq 0$ and $\tilde{Q}(x) \leq 0$ and
$$\tilde{Q}(x)\dot{x} = \nabla_x \tilde{P}(x) + \tilde{G}(x)u \tag{2}$$
describe the dynamics (1) (procedure for finding such pair is given in [22]). Such $\tilde{P}$ and $\tilde{Q}$ are called *admissible pairs* for (1).
2) $\tilde{G}(x)$ is Integrable.

The control objective is to stabilize the system at the equilibrium point (x^*, u^*) satisfying

$$\nabla_x \tilde{P}(x^*) + \tilde{G}(x^*)u^* = 0 \tag{3}$$

Proposition 1: Consider the system (1) in BM form (2) satisfying assumption 1. Then the system is passive with input u, output given by $y_{PB} = -\tilde{G}(x)^T \dot{x}$ and storage function $\tilde{P}$.

Proof: Time differential of $\tilde{P}$ is given by

$$\begin{aligned} \dot{\tilde{P}} &= (\nabla_x \tilde{P})^T \dot{x} \\ &= \dot{x}^T \tilde{Q}\dot{x} + u^T y_{PB} \\ &\leq u^T y_{PB}, \end{aligned} \tag{4}$$

where y_{PB} is given by

$$y_{PB} = -\tilde{G}(x)^T \dot{x} \tag{5}$$

which is referred as power balancing (shaping) output [16].

Proposition 2: The power balancing output y_{PB} given in (5) is integrable.

Proof: From Assumption 2, we have that $\tilde{G}(x)$ is integrable, Poincaré's Lemma ensures the existence of a function $\Gamma(x) : \mathbb{R}^n \to \mathbb{R}^n$ such that

$$\dot{\Gamma} = -\tilde{G}(x)^\top \dot{x} \tag{6}$$

using (5) we conclude the proof.

To achieve the control objective, we need to find a new storage function P_d of the closed loop system such that

$$\tilde{Q}\dot{x} = \nabla_x P_d \quad \text{and} \quad x^* = \arg\min_x P_d \tag{7}$$

The closed loop potential function P_d is difference of power function $\tilde{P}$ and power supplied by the controller. In [23], the power supplied by controller is found by solving PDE's. Here, we adopt the procedure without solving PDE using the power balancing outputs of the system which is similar to given in [24]–[26], where they have used for energy shaping for a class of mechanical systems. Also recently in [27] similar idea is used for systems in the port-Hamiltonian form, using the Hamiltonian as the systems stored energy. By exploiting the Assumption 2, in Proposition 2 we have proved that the power balancing output is integrable. Using this the desired closed loop potential function P_d is constructed in the following way

$$P_d = k\tilde{P} + \frac{1}{2}||\Gamma(x) + a||^2_{k_I} \tag{8}$$

where $k > 0$, $a \in \mathbb{R}^m$, $k_I \in \mathbb{R}^{m \times m}$ with $k_I > 0$. And further a is chosen such that (7) is satisfied, which implies

$$\nabla_x P_d(x^*) = 0 \qquad \nabla_x^2 P_d(x^*) \geq 0 \tag{9}$$

which upon solving gives

$$a := k k_I^{-1} \tilde{G}^\dagger(x^*) \nabla_x \tilde{P}(x^*) - \Gamma(x^*) \tag{10}$$

where $\tilde{G}^\dagger$ represents pseudoinverse of $\tilde{G}$.

Proposition 3: Consider the system (1) satisfying the assumptions 1 and 2. We define the mapping $u : \mathbb{R}^n \to \mathbb{R}^m$

$$u := \frac{1}{k}\left(v + \alpha \tilde{G}^\top \dot{x} - k_I(\Gamma(x) + a)\right). \tag{11}$$

where $\alpha > 0$, $\nabla\Gamma(x) := -\tilde{G}(x)$. Then system (1) in closed loop is passive with storage function P_d (8) satisfying (7), input v and output y_{PB}. Further with $v = 0$ the system (1) is stable with Lyapunov function $P_d(x)$ and x^* as stable equilibrium point. Furthermore, if $y_{PB} = 0 \implies \lim_{t\to\infty} x(t) \to x^*$, then x^* is asymptotically stable.

Proof: The time derivative of closed loop potential function (8) is

$$\begin{aligned} \dot{P}_d &= k\dot{\tilde{P}} + y_{PB}^T k_I(\Gamma(x) + a) \\ &\leq y_{PB}^T [ku + k_I(\Gamma(x) + a)] \\ &\leq y_{PB}^T v - \alpha y_{PB}^T y_{PB} \\ &\leq y_{PB}^T v, \end{aligned}$$

where we used equations (4),(5),(11) in arriving at the result. This proves that the closed loop is passive with storage function P_d (8), input v and output y_{PB}. Further for $v=0$ we have

$$\dot{P}_d \leq -\alpha y_{PB}^T y_{PB}$$

and at equilibrium

$$u^* = -\frac{k_I}{k}\left(\Gamma(x^*)+a\right). \tag{12}$$

Finally from (10) and (12) we can show that (x^*, u^*) satisfy (3). This concludes the system (1) is asymptotically stable with Lyapunov function P_d and x^* as equilibrium point [28].

III. Control of HVAC subsystems

In typical building HVAC systems, we have Air-side and Water-side HVAC subsystems. While air side focusses on delivering conditioned air to the zones, water side is responsible for various heat exchanging operations. In this section we apply the proposed approach on examples of both air-side and water-side subsystems.

A. Building Temperature control

The building thermal model of a multi-zone building based on first principles such as energy and mass balance equations will lead to coupled partial differential equations. There are several difficulties associated with such kind of models in terms of prediction and control design purpose. Since the building is an interconnected system with individual zones as its subsystems and interactions between these zones can occur due to conduction, convection and radiation. In this paper, we assume that the interaction between different zones occurs only through conduction and contribution due to convection and radiation is negligible. The supplied air to the zone is modulated at the Variable Air Volume (VAV) boxes by changing the flow rate and temperature of air through dampers and reheat coils. In this section, a different view-point to modeling and regulation of temperature in a multi-zone building using power is presented. The advantage of using Brayton-Moser framework is that it naturally describes the dynamics of systems in terms of measurable quantities. In the case of building systems the individual zone temperatures are easily measurable and the controller designed can be used to improve the transient and the steady state response.

1) Building Zone Model: A building zone model is constructed [10] by combining lumped parameter models of thermal interaction between zones separated by a solid surface (e.g walls). A lumped parameter model of combined heat flow across a surface is modeled as RC-network, with current and voltage being analogous of heat flow and temperature. In this modeling framework, the capacitances are used to model the total thermal capacity of the wall, and the resistances are used to represent the total resistance that the wall offers to the flow of heat from one side to other.

The thermal dynamics of a multi-zone building are given by:

$$C_i\dot{T}_i = \sum_{j\in\mathbb{N}_i}\frac{(T_j-T_i)}{R_{ij}} + u_i + \underbrace{\frac{(T_\infty - T_i)}{R_{i0}}}_{Q_i} \tag{13}$$

where $\mathbb{N}_i$ denotes all resistors connected to the ith capacitor (includes zone and surface capacitances), T_∞ is the ambient temperature. u_i is the heating/cooling generation input to the ith zone and Q_i is the external heat input due to ambient and is nonzero only for the zone nodes. In order to illustrate the proposed idea of power based modeling and regulation of building systems, we consider the dynamics of simple case of a two-zone building separated by a surface [29], where the surface is modeled as a 3R2C network is shown in Figure (1). The dynamics of the system is given by

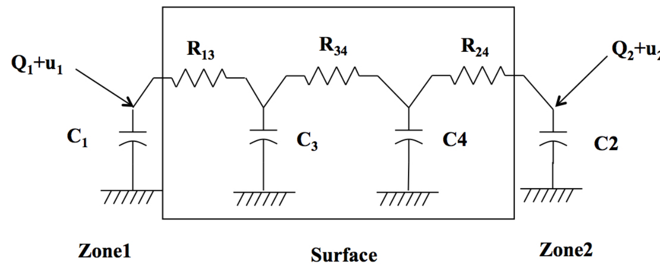


Fig. 1: Two zones separated by surface and lumped RC network model.

$$\begin{aligned}
C_1\dot{T}_1 &= \frac{T_3-T_1}{R_{31}} + Q_1 + u_1 \\
C_2\dot{T}_2 &= \frac{T_4-T_2}{R_{42}} + Q_2 + u_2 \\
C_3\dot{T}_3 &= \frac{T_1-T_3}{R_{31}} + \frac{T_4-T_3}{R_{34}} \\
C_4\dot{T}_4 &= \frac{T_2-T_4}{R_{42}} + \frac{T_3-T_4}{R_{34}}
\end{aligned} \tag{14}$$

Here T_1, T_2 are zone temperatures and T_3, T_4 are surface temperatures.

The above system of equations (14) can be written in the Brayton-Moser form (2) with $x=\left[T_1,T_2,T_3,T_4\right]^\top$, and

$$\begin{aligned}
P(x) &= \frac{(T_3-T_1)^2}{2R_{31}} + \frac{(T_4-T_2)^2}{2R_{42}} + \frac{(T_3-T_4)^2}{2R_{34}} \\
&\quad + \frac{(T_\infty-T_1)^2}{2R_{10}} + \frac{(T_\infty-T_2)^2}{2R_{20}}. \qquad (15) \\
Q(x) &= \mathrm{diag}[-C_1,-C_2,-C_3,-C_4] \text{ and} \\
G(x) &= \begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\end{bmatrix}^\top.
\end{aligned}$$

It is easily verified $P(x)$, $Q(x)$ and $G(x)$ defined in (15) satisfy assumption 1 and 2. From Proposition 1, system (14) is passive with input $u=[u_1,u_2]^\top$, power balancing output $y=[\dot{T}_1,\dot{T}_2]^\top$ and storage function $P(x)$, further from Proposition 2, we have $\Gamma(T)=[T_1,T_2]^\top$.

Control objective: The control objective is to stabilize a given equilibrium point $[T_1^*, T_2^*]$ satisfying (3) where

$$\begin{aligned}
u_1^* &= -\left(\tfrac{(T_3^*-T_1^*)}{R_{31}} + \tfrac{(T_\infty-T_1^*)}{R_{10}}\right) \\
u_2^* &= -\left(\tfrac{(T_4^*-T_2^*)}{R_{42}} + \tfrac{(T_\infty-T_2^*)}{R_{20}}\right)
\end{aligned} \tag{16}$$

B. Controller design

Proposition 4: Consider the closed loop storage function defined in (8) with $k_I = \text{diag}(k_1, k_2)$ and $a = [a_1, a_2]^\top$. P_d defined in (8), takes the form

$$P_d = kP + \frac{k_1}{2}(T_1 + a_1)^2 + \frac{k_2}{2}(T_2 + a_2)^2 \quad (17)$$

(a) for $a_1 = -\frac{k}{k_1}u_1^* - T_1^*$, $a_2 = -\frac{k}{k_1}u_2^* - T_2^*$, P_d is positive definite and has a minimum at $[T_1^*, T_2^*]$.

(b) further with the state feedback controller (11)

$$\begin{aligned} u_1 &= -\tfrac{\alpha}{k}\dot{T}_1 - \tfrac{k_1}{k}\left(T_1 - T_1^* - \tfrac{k}{k_1}u_1^*\right) \\ u_2 &= -\tfrac{\alpha}{k}\dot{T}_2 - \tfrac{k_2}{k}\left(T_2 - T_2^* - \tfrac{k}{k_2}u_2^*\right). \end{aligned} \quad (18)$$

If the tuning parameters α, k, k_1, k_2 are nonnegative, then $[T_1^*, T_2^*]$ is asymptotically stable equilibrium of the closed loop system with P_d as Lyapunov function.

Proof:

We need to choose a such that $\nabla P_d(x^*) = 0$ and $\nabla^2 P_d(x^*) \geq 0$ at the desired equilibrium. Therefore, proof of (a) directly follows from (10) and (12). The proof of (b) follows from Proposition 3. It can also be proved by taking the time differential of the Lyapunov functional P_d defined in (17) as shown below

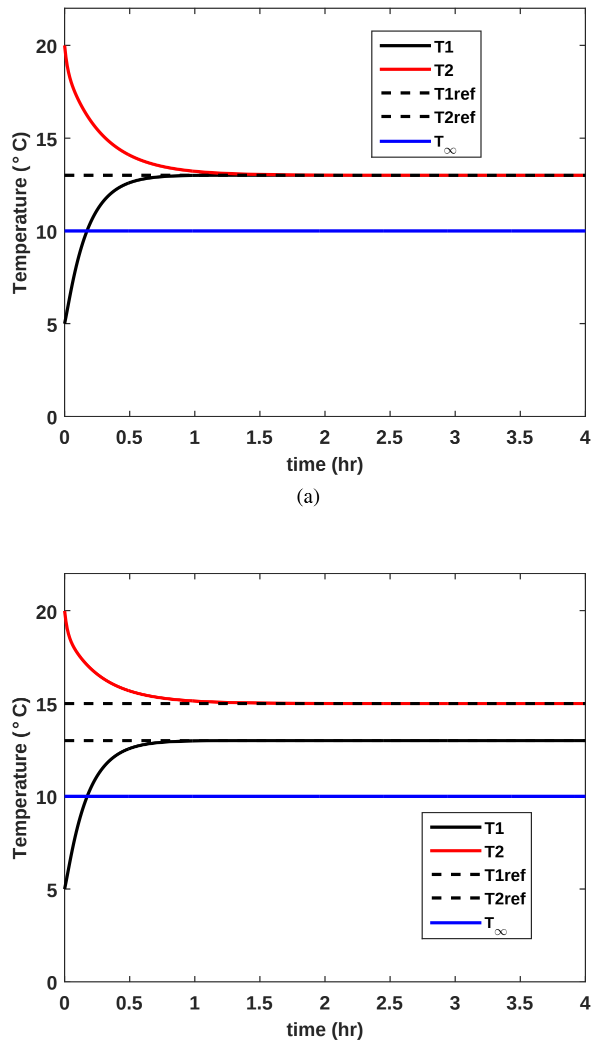
$$\begin{aligned} \dot{P}_d &= k\dot{P} + k_1(T_1 + a_1)\dot{T}_1 + k_2(T_2 + a_2)\dot{T}_2 \quad (19) \\ &= k(\dot{T}_1 u_1 + \dot{T}_2 u_2) + k_1(T_1 + a_1)\dot{T}_1 + k_2(T_2 + a_2)\dot{T}_2 \\ &= \dot{T}_1(ku_1 + k_1(T_1 + a_1)) + \dot{T}_2(ku_2 + k_2(T_2 + a_2)) \end{aligned}$$

Using u_1 and u_2 from (18) the resulting equation (19) becomes

$$\frac{d}{dt}P_d \leq -\alpha(\dot{T}_1{}^2 + \dot{T}_2{}^2) \leq 0.$$

The controller obtained is a PI controller with respective to power balancing outputs. The controller needs model information to compute u^*, but the system attains stability for error in u^*. The analysis provided uses zone heating/cooling as input, but the proposed approach can be easily extended to more general model where the zone mass flow rate is the control variable [29].

Simulations were conducted on the simple two-zone model, in order to show the effectiveness of the proposed approach. Different operating conditions are considered, where the zones temperatures have same and different set points and with different outside air temperatures. The parameters used for the simulation can be found in [29]. The objective is to regulate the zone temperatures such that $T_1 = T_1^*$, $T_2 = T_2^*$. Figure 2 shows the case where the individual zones are subjected to constant ambient temperature with same and different set points. The controller effectiveness in terms of transient and steady state performance is verified in regulating the zone temperatures to their corresponding set points. The important note is that there is no overshoot in the time response of states before settling to the target values, which shows the effectiveness of controller compared to energy based controllers. In order to study the actual situation, we consider a time varying ambient. Figure 3 shows the case where the individual zones are subjected to time varying sinusoidal ambient temperature ($T_\infty = 5\sin(2\pi t/T) + 5°C, T = 24$hrs [10]) with same and different set points, the controller performs reliably under different ambient temperatures.



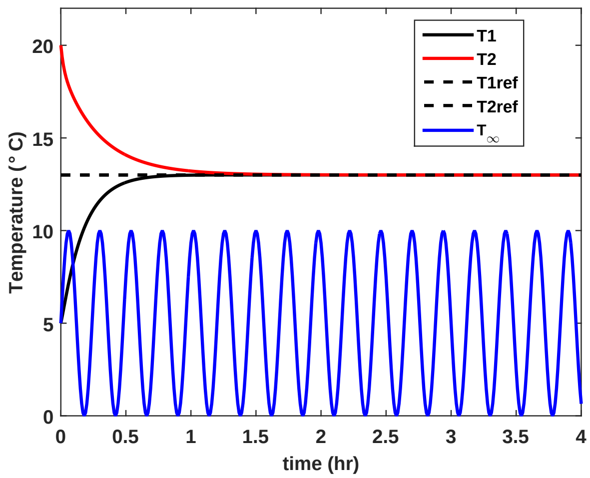
(a)

(b)

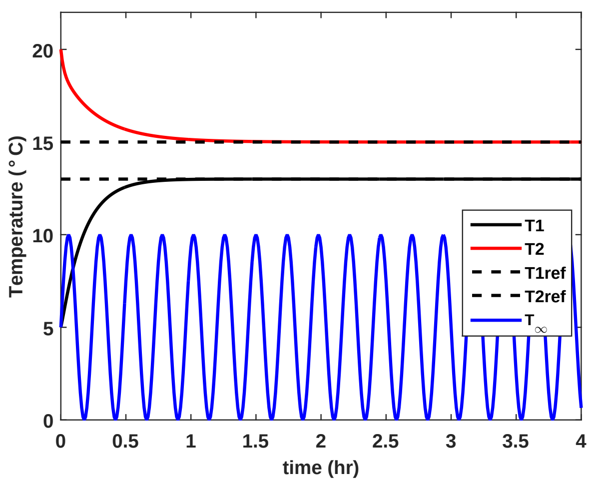
Fig. 2: Zone temperature for constant ambient temperature a) Same reference b) Different reference.

C. Heat exchanger

Heat exchangers are one of the most important HVAC subsystems which transfer heat from one medium (water/air) to another (water/air). The effectiveness of heat exchangers strongly influences the thermal performance of building systems. To illustrate the proposed approach, we consider a water-to-water heat exchanger where heating is accomplished either by geothermal or solar energy. We consider a simple tube-shell water-to-water heat exchanger model given in [21], and the corresponding schematic is shown in Fig. 4. The inlet and outlet temperatures of cold stream are given by T_{ci},T_{co} whereas the corresponding temperatures on hot-stream side



(a)



(b)

Fig. 3: Zone temperature for varying ambient temperature a) Same reference b) Different reference.

are denoted by T_{hi}, T_{ho}, respectively. The control variables are the volumetric flow rates denoted by f_c,f_h; the thermal capacities for the cold and hot stream are denoted by C_c,C_h; and the heat transfer in the system is modeled by a thermal conductance G_{hc}. The differential equations governing the heat exchanger system are given by

$$\begin{aligned} C_c\dot{T}_{co} &= -G_{hc}(T_{co} - T_{ho}) + \gamma_c(T_{co} - T_{ci})f_c \\ C_h\dot{T}_{ho} &= G_{hc}(T_{co} - T_{ho}) + \gamma_h(T_{ho} - T_{hi})f_h \end{aligned} \tag{20}$$

The system of equation (20) can be written in Brayton-Moser form (2) with $x = [T_{co}, T_{ho}]^\top$ and

$$\begin{aligned} P(x) &= \frac{G_{hc}}{2}(T_{co} - T_{ho})^2 \\ Q(x) &= \text{diag}(-C_c, -C_h) \text{ and} \\ G(x) &= \begin{bmatrix} -\gamma_c(T_{co} - T_{ci}) & 0 \\ 0 & -\gamma_h(T_{ho} - T_{hi}) \end{bmatrix}. \end{aligned} \tag{21}$$

It can easily be verified that $P(x)$, $Q(x)$ and $G(x)$ in (21) satisfy Assumptions 1 and 2. From Proposition 1, the system

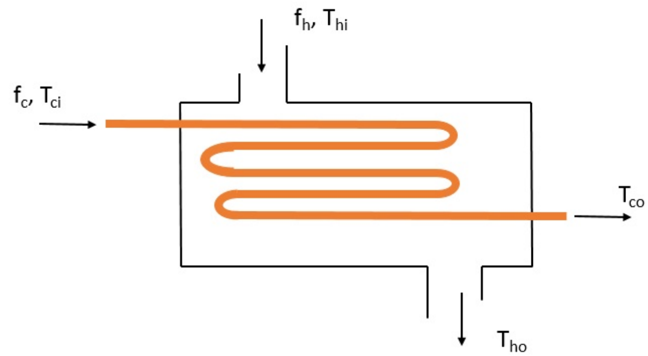


Fig. 4: Heat exchanger model.

defined in (20) is passive with storage function $P(x)$ in (21), input $u = [f_c, f_h]^\top$ and output

$$y = [\gamma_c(T_{co} - T_{ci})\dot{T}_{co}, \gamma_h(T_{ho} - T_{hi})\dot{T}_{ho}]^\top. \tag{22}$$

Further from Proposition 2, we have

$$\begin{aligned} \Gamma_1 &= \frac{\gamma_c}{2}(T_{co} - T_{ci})^2 \\ \Gamma_2 &= \frac{\gamma_h}{2}(T_{ho} - T_{hi})^2 \end{aligned} \tag{23}$$

Control objective: The control objective is to stabilize system (20) at operating point $[T_{co}^*, T_{ho}^*, u_1^*, u_2^*]$ satisfying (3) that is

$$u_1^* = \frac{G_{hc}(T_{co}^* - T_{ho}^*)}{\gamma_c(T_{co}^* - T_{ci})} \quad u_2^* = -\frac{G_{hc}(T_{co}^* - T_{ho}^*)}{\gamma_h(T_{ho}^* - T_{hi})} \tag{24}$$

Similar to Proposition 4, in this example, using Proposition 3 we can show that system (21) in the closed-loop with feedback controller

$$\begin{aligned} u_1 &= -\frac{\alpha}{k}\gamma_c(T_{co} - T_{ci})\dot{T_{co}} - \frac{k_1}{k}\left(\Gamma_1 - \Gamma_1^* - \frac{k}{k_1}u_1^*\right) \\ u_2 &= -\frac{\alpha}{k}\gamma_c(T_{ho} - T_{hi})\dot{T_{ho}} - \frac{k_2}{k}\left(\Gamma_2 - \Gamma_2^* - \frac{k}{k_2}u_2^*\right) \end{aligned}$$

is asymptotically stable at equilibrium $[T_{co}^*, T_{ho}^*]$ with Lyapunov function (8) defined with $k_I = \text{diag}(k_1, k_2)$ and $a = -kk_I^{-1}u^* - \Gamma(x)^*$. The objective is to achieve a desired outlet temperature of cold stream $T_{co}^* = 80°C$. This gives a desired equilibrium (T_{co}^*, T_{ho}^*), where T_{ho}^* is determined by χ and T_{co}^*, the admissible equilibrium set χ is given by

$$\chi = \{(T_{co}, T_{ho}) \in \mathcal{S} | G_{hc}(T_{co} - T_{ho}) + \gamma_h(T_{ho} - T_{hi})f_h = 0\}$$

and $\mathcal{S} = \{(T_{co}, T_{ho}) \in \mathbb{R}^2 | T_{co} > T_{ci}\}$. The parameters values used for the simulation are found in [30]. From Fig. 5, it can been seen that the desired outlet temperature of cold stream is attained and lies on the equilibrium manifold, which shows the performance of the controller in regulating the temperature.

IV. Conclusions

In this paper, a new paradigm is presented for synthesizing HVAC control of building systems using power shaping approach that exploits passivity property of the system.

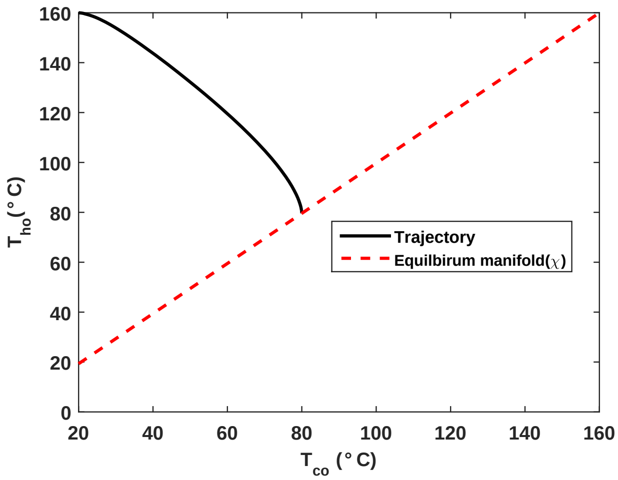


Fig. 5: Closed loop trajectory.

The building systems are getting increasingly complex and model-based control approaches face huge challenges due to inherent uncertainties and modeling inaccuracies. The passivity-based approach can offer inherent robustness to such uncertainties and modeling inaccuracies as long as input-output passivity property is unaltered. The power shaping paradigm has been found successful in other applications but has not been used in building systems control. This paper presented two numerical examples to demonstrate the applicability of power-shaping approach. It was shown that dynamics of HVAC systems can be transformed into the Brayton-Moser form and then power-shaping methodology can be used to design an effective controller. The follow-on work will extend the methodology to include additional dynamics such as occupancy, solar radiation, and also focus on design and implementation of the control system on an experimental building energy systems laboratory.

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