Abstract—This paper proposes a framework for reactive
goal-directed navigation without global positioning facilities
in unknown environments. A mobile sensor network is used
for localization of regions of interest for path planning of an
autonomous mobile robot in the absence of global positioning
facilities. The underlying theory is an extension of a generalized
gossip algorithm that has been recently developed in a language-
measure-theoretic setting. The gossip algorithm has been used
to propagate local decisions of target detection over a mobile
dense network and thus, it generates a belief for the target
detected over the network. The proposed concept has been
validated through numerical experiments with a mobile sensor
network and a point mass robot.

1. INTRODUCTION

Autonomous robots are becoming ubiquitous and are en-
visaged to play an increasingly important role in both civilian
and military applications. While operating in unknown and
unstructured environments, they often have limited or unreli-
able long-range communication and GPS capabilities due to
constraints on energy requirements and (or) adversarial envi-
ronment. However, with the recent advances in sensing and
low-complexity signal processing algorithms, sensors can lo-
cally detect regions of interest with high degree of accuracy,
which reduces the communication overhead significantly. But
it limits event awareness which limits performance in the
network capacity. Consider, for example, a search and rescue
operation which requires sequential collaboration between
the ‘search agents’ (agents with limited sensing abilities)
and the ‘rescue agents’ (agents capable of rescuing any
sensed target in a region of interest). The efficacy of such
missions in the network capacity would depend on how
quickly the network of ‘search agents’ can react to the sensed
targets of interest and guide a ‘rescue agent’ to the target
of interest under the constraints of limited communication
and global positioning. The goal of this paper is to present
a framework for navigation of an autonomous agent in an
unknown and GPS-denied environment with the help of a
(possibly mobile) sensor network which serves the two-fold
purpose of target localization and generating the way-points
for the autonomous robot.

A lot of work has recently been reported on source seeking
in sensor fields [1] [2] [3] [4]. The objective of such prob-
lems is to determine the minimum of an unknown signal field
(the possible location of the source) using a stochastic gra-
dient descent algorithm. Authors in [2] [4] present a multi-
agent coordination framework for estimation of the peaks of
sensor field. However, the agents have to communicate their
sensor measurements and an artificial potential function is
required to estimate the gradient of the sensor field. Several
attempts have been made in literature to make use of static
sensor networks to guide a robot [5], [6]. In [6], a pseudo-
gradient calculated based on sensor readings in a static sensor
network is used for localization and directed navigation of an
autonomous robot in unknown environments. However, the
algorithm presented can’t be trivially extended to navigation
using a mobile sensor network. Mobile sensor networks have
potential advantage over their static counterparts in terms of
coverage and time-criticality [7].

The current work builds on a recent work on distributed
decision propagation in mobile ad-hoc sensor networks pre-
sented in [8] where the proximity network of an agent is
modeled as a probabilistic finite state automaton (PFSA). An
‘agent measure function’ is then defined (based on the re-
cently reported language measure theory [9] [10]) for all the
agents in the network which signifies its ‘level of awareness’
regarding a locally sensed ‘target’ in the operational area. In
the current work, the agent measure function generated by a
mobile sensor network is used to guide an autonomous robot
through an unknown and unstructured environment. The
current framework has the following potential advantages
over those reported in literature:

• The algorithm only requires exchange of local decisions
  about sensed targets, and not the actual sensor mea-
surements. This has the potential to significantly reduce
  communication overhead and makes the network more
  robust to communication flips.

• No artificial potential function is required to guide
  an agent to the locally detected goal; a gradient is
  automatically generated by the agent measure function
  which is maximized at the location of sensed target.

This is different from other works present in current
literature [5] in the sense that not all sensors in the network
detect the target; an awareness about the presence of a
local target is developed via gossip and it is fed back to
a continuous time controller of a robot to find a path. A
sampling-based algorithm is used to tackle the dynamics of
the robot at a lower continuous control level.

It is noted that the term agent is often interchangeably
used for mobile sensor and shouldn’t be confused for robot.

2. Background on Distributed Decision
Propagation in Proximity Networks

This section briefly summarizes the concept of real
measure of probabilistic regular languages generated by a
PFSA [10] [9] followed by the formulation of generalized
gossip algorithm presented in [8].

A. Basic Notions of Language-Measure Theory

For brevity, the concept of real measures have been
restricted to irreducible Markov Chain. Interested reader is
referred to [9] [10] for further details.

Definition 2.1 (Real Measure of Irreducible Markov Chain)
Let a stationary Markov chain be denoted by the three-tuple
\((Q, \Pi, \chi)\), where \(Q\) is the set of states; the state transition
function \(\Pi : Q \times Q \rightarrow [0,1]\) represents the \(|Q| \times |Q|
stochastic matrix for the Markov chain (\(|Q|\) represents the
cardinality of the set of states); and \(\chi : Q \rightarrow \mathbb{R}\) is the
vector-valued characteristic function that assigns a signed
real weight to each state. A real measure \(\nu_i(\theta)\) for state \(i\) is
then defined as

\[
\nu_i(\theta) \triangleq \sum_{k=0}^{\infty} \theta(1-\theta)^k \Delta_i \Pi^k \chi,
\]

where \(\theta \in (0,1)\) is a user specified parameter and \(\Delta_i\) is
defined as a \(1 \times |Q|\) vector \([\delta_{i1}, \delta_{i2}, \ldots \delta_{i|Q|}]\) which is given as
\(\delta_{ij}=1,\text{ if } i=j,\text{ else zero.}\) The expression for the measure
in Equation 1 can be expressed as : \(\nu(\theta) = \theta(1 - (1 - \theta)\Pi)^{-1} \chi\)
The inverse is guaranteed to exist for \(\theta \in (0,1)\).

\(\Delta_i \Pi^k\) represents the state probability vector at an instant
\(n\) time-steps in the future for a Markov process beginning in
state \(i\) and the expected value of the characteristic function
is given by \(\Delta_i \Pi^n \chi\).

B. Background of Distributed Decision Propagation

This subsection briefly describes the formulation of the
generalized gossip policy in the context of proximity net-
works proposed in [8]. Proximity network [11] is a particular
formulation of time-varying mobile-agent networks, inspired
from social networks where only proximal agents communi-
cate at any given time epoch [12].

In the present context, proximal agents exchange informa-
tion related to their beliefs regarding the environment. After
the expiry of a message lifetime \(L_m\), agents possibly update
their beliefs based on their own observation and messages
from other agents. There are two time-scales involved in this
problem setup. In contrast to the faster time-scale \((t)\) of agent
motion, the algorithm for updating the agents’ beliefs runs on
a (possibly) slower time-scale (denoted by \(\tau\)). The time-scale
for updating the belief is chosen to be slower as it allows
for sufficient interactions among the agents, especially if the
density of agents is low. To capture temporal effects in a
realistic setting, \(L_m\) should be appropriately chosen based
on other network parameters.

With this setup, let a time-dependent (in the slow-scale \(\tau\)
graph be denoted as \(G\) and a few related terms are defined
as follows.

Definition 2.2 (Adjacency Matrix [13]) The adjacency ma-
trix \(A\) of the graph \(G\) is defined such that its element \(a_{ij}\)
is unity if the agent \(i\) communicates with the agent \(j\) in
the time period of \(L_m\), else zero. To eliminate self-loops, each
diagonal element of the adjacency matrix is constrained to
be zero.

Definition 2.3 (Laplacian Matrix [13]) The Laplacian ma-
trix \((\mathcal{L})\) of a graph \(G\) is defined as: \(\mathcal{L} = \mathcal{D} - \mathcal{A}\) where
the degree matrix \(\mathcal{D}\) is a diagonal matrix with \(d^i\) denoting the
degree of node \(i\).

Definition 2.4 (Interaction Matrix [13]) The agent interac-
tion matrix \(\Pi\) is defined as: \(\Pi = I - \beta \mathcal{L}\)

The generalized gossip strategy involves two characteris-
tic variables associated with each agent, namely the state
characteristic function \(\chi\) and the agent measure function \(\nu\).
\(\chi \in \{0,1\}\) signifies whether an agent has detected a target
(\(\chi = 1\)) or not (\(\chi = 0\)). \(\nu \in [0,1]\) signifies the level of
awareness of an agent regarding the presence of a target
in the surveillance region. It is noted that, \(\Pi, \nu\) and \(\chi\) are
functions of the slow time-scale \(\tau\). In the above setting, a
decentralized strategy for measure updating in the mobile-
agent population is introduced below in terms of a user-
defined control parameter \(\theta \in (0,1]\).

\[
\nu_i(\theta)|_{\tau+1} = (1-\theta) \sum_{j \in \{i\} \cup Nb(i)} \Pi_{ij}|_{\tau} \nu_j(\theta)|_{\tau} + \theta \chi_i|_{\tau}
\]

where \(Nb(i)\) denotes the set of agents that communicate with
the agent \(i\) during the time span between \(\tau\) and \(\tau+1\). It is
noted that while computing the future (awareness or belief)
measure of an agent, the parameter \(\theta\) controls the trade-off
between the effects of current self-observation and current
measures of all agents. In the vector notation, the dynamics
can be expressed as: \(\nu(\theta)|_{\tau+1} = (1-\theta)\Pi|_{\tau} \nu(\theta)|_{\tau} + \theta \chi|_{\tau}\).
Thus, this policy is simply a gossip algorithm with varying
input \(\chi|_{\tau}\) and varying network topology represented by
\(\Pi|_{\tau}\). The memory of a past input fades as a function of
the parameter \(\theta\). Due to this notion, the above policy can
be called a generalized gossip algorithm with \(\theta\) as the
generalizing parameter.

In the following sections, agent measure function is often
referred to as belief.
3. Problem Formulation

This section formulates the problem of path planning for an autonomous robot in the absence of GPS. For simplicity of exposition, we make certain assumptions to unambiguously present the efficacy of the proposed framework for reactive navigation in the absence of GPS, which are outlined below.

1) An autonomous robot can locally estimate relative positions of mobile sensors using state-of-art positioning techniques in sensor networks [14].
2) Mobile sensors and the autonomous robot are locally able to coordinate for collision avoidance.
3) Communication of the robot with other mobile sensors is considered in the time scale $T \gg \tau$.

Under these major assumptions, we consider the case of a set of mobile sensors performing surveillance in a region, where the task is to detect targets in a given region. For simplicity, the target (i.e., the goal for the autonomous robot) is modeled as a local region of interest in the surveillance region such that only a few sensors that search areas within the region of interest have a non-zero probability of detecting it. For clarity, a simplistic model for target detection is followed which is described next. A region of interest is modeled as a map for probability of detection of a target. Let the probability of detection of a target be denoted by $P_D$, which attains the maximum at the center of the target’s physical location and decays to zero linearly with distance from the center in a radially symmetric manner. A region of interest is then characterized by the following parameters:

- The maximum probability of detection of the target, $P_{D_{\text{max}}}$
- The effective radius $(r_{hs})$ of the circular region within which $P_D > 0.5$.

In [8], a distributed decision propagation algorithm has been proposed for dissemination of the sensed target throughout the mobile sensor network. The aim of the current work is to develop a distributed navigation algorithm to help guide an autonomous robot to the detected region of interest (i.e., goal for the autonomous robot) where no one is aware of the sensed location of target and there is no GPS. Since the robot has only a finite sensing and communication radius, it can only be aware of the local belief in the network. The problem of reactive navigation to a locally detected target is then reduced to the recursive estimation of a sequence of way-points which the robot can follow to finally reach the goal. Even though the broad scope of this research is to allow a robot with any complicated dynamics find a feasible trajectory, for simplicity, this paper is limited to a point mass robot.

4. Proposed Approach

Under the framework presented in [8] and briefly explained in section 2, we present an algorithm which guarantees a unique maxima and a gradient towards the same in the proximity network. The idea is that if the autonomous robot moves in a way so that its belief (based on the belief of its nearby mobile sensors) monotonically improves (or increases) with movement, then under the condition that the belief of the network is maximized at the physical location of the goal, the robot will eventually reach the goal. Under the constraints of limited communication and sensing horizon, the robot has access to belief of only the local mobile sensors. However, due to the presence of a gradient towards the goal, the robot is able to estimate a waypoint where the belief is greater than its current belief. To this end, the robot learns an implicit correspondence between a geographical location and the belief in the network by using a multiple regression framework. The maximum of the implicit surface is the waypoint the robot moves to, over a certain time horizon till the next communication with the network is established. This is achieved by following a feasible trajectory obtained by sampling from the local configuration space using RRTs. These steps are recursively followed till the robot reaches the sensed region of interest i.e., the goal. The idea is similar to the commonly studied receding horizon motion planning framework, where a reactive plan is followed by the robot over a finite time horizon as a reaction to real-time information.

A. Decentralized Gossip for Decision Propagation

Based on the framework of generalized gossip algorithm, this subsection presents an algorithm which creates a bias in the proximity network of the mobile sensors towards the sensed region of interest. The idea is based on optimal control theory of a PFSA [10] [15]. Under this umbrella, the belief of every sensor is maximized by averaging over the set of neighbors that have belief greater than the sensor. In the original gossip strategy (see equation 2), a sensing agent is influenced by all its neighbors. However, to maximize its measure, an agent can follow a strategy where it is only influenced by neighbors that have a higher belief than its own belief. This strategy is succinctly presented in Algorithm 1. The key point is that the elements of the interaction matrix corresponding to agents with a lower measure are made zero. However, to keep the interaction matrix stochastic, those elements are adjusted as a self-loop to the agent (see steps 5 through 11 in Algorithm 1). Based on the results in [10] [15], this strategy ensures a maximum in the belief network at the goal region for the autonomous agent and at the same time, it creates a gradient towards the same. This biased approach ensures that a mobile sensor which is closer to the sensed region of interest will have a higher belief as compared to those further away from it.

B. Implicit Surface based Interpolation for Navigation

Under the assumption that the robot can localize mobile sensors in its neighborhood [14], beliefs of the mobile sensors in the robot’s neighborhood are used to learn an
correspondence between the local physical locations and RBFs are used [16]. The function $F(x)$ is then, formally stated as follows: Given the approximate estimates $x^i_R$, $i = 1, 2, \ldots, M$, using localization techniques in sensor network. The interpolation problem is then, formally stated as follows: Given the approximate locations of the neighbors of the robot, $\{x^i_R \in \mathbb{R}^2, i = 1 \text{ to } M\}$ and their corresponding beliefs $\{\nu_i \in \mathbb{R}\}$, a function $\mathcal{F} : \mathbb{R}^2 \rightarrow \mathbb{R}$ is estimated, such that it satisfies the boundary constraints

$$\mathcal{F}(x^i_R) = \nu_i, \quad i = 1, 2, \ldots, M$$

where $\mathcal{F}(x^R)$ has the following form

$$\mathcal{F}(x^R) = \sum_{i=1}^{M} w_i \phi(||x^R - x^i_R||)$$

Let us assume that $\{x^R_{\max}(T_1), x^R_{\max}(T_2), \ldots, x^R_{\max}(T_n)\}$ is the sequence of way-points estimated by the robot in the slow time scale at instants $T_1, T_2, \ldots, T_n$. Then, if the robot moves in a way such that $\{v^\ast_{\max}(T_1), v^\ast_{\max}(T_2), \ldots, v^\ast_{\max}(T_n)\}$ is a monotonically increasing set, then the following will hold:

$$\text{dist}\left(\lim_{n \to \infty} x^R_{\max}(T_n), x^S\right) < \epsilon$$

In this work, an inverse multi-quadric form of $\phi$ was chosen. Specifically, $\phi(||x^R - x^i_R||)$ is chosen to be, $1/\sqrt{\tau^2 + \sigma^2}$, where $r = ||x^R - x^i_R||_2$ and $\sigma$ is chosen as $k/\theta$. As it was explained before in section 2, $\theta$ is the generalizing parameter for the gossip algorithm. In order to determine $\{w_i, i = 1 \text{ to } M\}$, a multiple regression algorithm is used. The procedure is succinctly presented in Algorithm 2.

C. RRT based Navigation

In the last step, the robot gets an estimate of the waypoint it should move to. Based on the current location and the waypoint found in the last step, a rapidly exploring random tree (RRT) is built in an anytime fashion to find a feasible trajectory for the robot. Under this framework, we assume that the robot can avoid the static obstacles by building collision-free trees [17] For completeness of the paper, RRT has been succinctly explained in Algorithm 3.

Algorithm 1 Distributed belief updating strategy for mobile sensors

1: while true do
2:   for all sensors ‘i’ in the network do
3:     if $Nb(i) \neq 0$ then
4:       $d_i = \text{CARD}(Nb(i))$
5:       $\ast\ast$ Begin Infinite Asynchronous loop $\ast\ast$
6:       $\ast\ast$ Query $\nu(\theta)_{ij} \ast\ast$
7:     if $\nu^j(\theta)|_{\tau} \leq \nu^i(\theta)|_{\tau}$ then
8:       $\Pi_{ij}|_{\tau} = \Pi_{ij}|_{\tau} + 1$
9:     if $\nu^j(\theta)|_{\tau} > \nu^i(\theta)|_{\tau} \& \Pi_{ij}|_{\tau} = 0$ then
10:    $\Pi_{ij}|_{\tau} = 1/d_i$
11:    $\Pi_{ii}|_{\tau} = \Pi_{ii}|_{\tau} - 1/d_i$
12: end if
13: end if
14: $\nu^i(\theta)|_{\tau} = (1 - \theta) \sum_{j \in \{i\} \cup Nb(i)} \Pi_{ij}|_{\tau} \nu^j(\theta)|_{\tau} + \theta \chi^i|_{\tau}$
15: end for
16: end while

Remark 4.1 Correctness: The plan will always give the robot a path to the sensed goal. This is argued by making some observations. Due to the biased gossip algorithm based on the optimal control of a weighted PFSA, it is ensured that there is a gradient towards the goal. Under the assumption of
bounded uncertainties in the localization estimates of sensors within its communication radius, the robot can always locate a way-point which has a higher belief (as found by the interpolation function) than its current belief. As the measure is maximized at the target location, so, as long as the robot moves in such a way that its measure (i.e., belief about the presence of a goal) monotonically increases, it will end up at the goal.

5. Results for an Example Problem

This section presents results of numerical experiments for an example problem of surveillance and reconnaissance which involves a mobile sensor network and an autonomous robot which needs to navigate to a target detected by the mobile network. We consider a surveillance example for a region of area $A$ performed by $N$ mobile sensors, where each mobile sensor has a communication radius $R_c$. The robot has a communication and sensing radius $R_s$. The individual mission of the agents is to detect any target and communicate this to their neighbors. The global mission objective of the sensor network is to direct a robot with greater capabilities to the sensed region with target for neutralization of threat or to deliver a service. For the simulation study, the parameters are chosen as: $A = 500^2$, $N = 150$, $R_s = 50$, and $R_c = 100$. For modeling of target, the value of $P_D$ was chosen to be 0.9 and $r_{hs}$ was chosen to be 20. The generalized gossip parameter $\theta$ was chosen to be 0.02. The velocity of the mobile sensors in the network was chosen to be 5 and the maximum velocity for the robot was 10. The mobile sensors are moving in the region with a 2-D random walk fashion with the constant velocity. A slower velocity for the mobile sensors might result in a slower information propagation but, it results in more stable local dynamics for the robot. Target is located at $[450, 250]$ while the robot is at $[1, 1]$ to begin with. $\epsilon$ (see equation 7) is chosen to be equal to $r_{hs}$.

The robot starts moving towards the goal as soon as its local neighborhood becomes aware of the target detection through gossip. Once the robot becomes aware of the detection, it makes use of the disseminated distributed belief about the target to find a path to the target. Figure 2 shows the implicit surface for correspondence between the agent measure function and a geographical location in its local neighborhood at different time instants based on the communication with the mobile sensors in its communication radius. Communication is re-established after the robot reaches the estimated waypoint corresponding to the local implicit surface. Note that the different surfaces are estimated in the slow time scale $T$. Figure 3(a) shows the monotonic increment in the belief at the estimated waypoint based on the local implicit surface with the movement of the robot. It can be seen in Figure 3(a) that as soon as the robot becomes aware of a sensed target (belief $> 0$), it is able to move in such a fashion that its awareness about the presence of the target monotonically increases and finally converges to its maximum value as it reaches the goal. The belief of the way-points could also be used as a measure for degree of

Algorithm 2 Navigation of the robot
1: while $\text{dist}(x_R, x_S) > \epsilon$ do
2:   Solve $\tilde{f}(x_R) = \sum_{i=1}^{M} w_i \phi(||x_R - x_i^R||)$ using boundary constraints $\{x_i^R, v_i\}, x_i^R \in \text{Nb}(R)$
3:   Use $\tilde{f}(x_R)$ to estimate $x_{\text{max}}^R = \arg \max_{x \in \text{Nb}(R)} \tilde{f}(x_R)$ and $v_{\text{max}} = \max_{x \in \text{Nb}(R)} \tilde{f}(x_R)$
4:   RRT($x_R, K, \Delta t, x_{\text{max}}^R$) /* For the function RRT, see Algorithm 3 */
5: end while

Algorithm 3 RRT
1: Input: $q_{\text{init}}, K, \Delta t, q_{\text{goal}}$
2: Output: Tree $G$ with a path $P$ from $q_{\text{init}}$ to $q_{\text{goal}}$
3: for $k = 1$ to $K$ do
4:   $q_{\text{rand}} \leftarrow \text{RandConf}()$
5:   $q_{\text{near}} \leftarrow \text{NearestVertex}(q_{\text{rand}}, G)$
6:   $q_{\text{new}} \leftarrow \text{NewConf}(q_{\text{near}}, q_{\text{rand}}, \Delta t)$
7:   Add vertex $q_{\text{new}}$ to $G$
8:   Add edge $(q_{\text{near}}, q_{\text{new}})$ to $G$
9: end for
10: $q_{\text{goal}} \leftarrow \text{NearestVertex}(q_{\text{goal}}, G)$
11: Retrace a path $P$ from $q_{\text{goal}}$ to $q_{\text{init}}$ over $G$.
12: return $P$

Fig. 2. The implicit surface estimated by the robot at three different time instants in its local neighborhood during navigation to an unknown goal location. The goal is at $[450, 250]$.
completion of the mission; convergence of the belief to the maximum value suggests mission completion. Figure 3(b) shows the monotonic convergence of the robot’s movement to the unknown goal location under the proposed framework. Figure 3(b) shows the inherent goal-directedness in the robot’s motion once it sniffs (i.e., belief > 0) the presence of the target. Figure 3(c) shows Euclidean distance between the robot and the goal in the slow time scale T. Figure 3(c) shows the actual trajectory found by the lower level continuous controller and followed by the robot between two consecutive way-points located at [327, 125] and [360, 205] (see the blue square and green circle in plates (b) and (c)). It is noted this is not the complete path as the initial path as the initial location of the robot is at [1, 1]. For clarity of presentation, only a part of the entire path has been shown.

6. CONCLUSIONS AND FUTURE WORK

The paper presents a framework of hierarchical planning, where the dynamic knowledge of the goal is propagated over a mobile sensor network, which is used as a feedback for a low-level continuous controller to find a feasible path for the robot. Using a controlled gossip algorithm and sequential estimation of the way-points locally, it is shown that the robot is capable of finding a path to the goal point. However, the efficacy of the proposed path planning algorithm is contingent on the accuracy of the localization techniques executed over the sensor network.

The following topics are recommended for future research.

1) Extension to planning in presence of multiple targets and multiple regions of interest.

2) Identification of an explicit relationship between imperfections in localization in the sensor network and the navigation of the robot.

REFERENCES


