Path Planning in GPS-denied Environments: A Collective Intelligence Approach

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***Abstract*— This paper proposes a framework for reactive goal-directed navigation without global positioning facilities in unknown environments. A mobile sensor network is used for localization of regions of interest for path planning of an autonomous mobile robot in the absence of global positioning facilities. The underlying theory is an extension of a generalized gossip algorithm that has been recently developed in a language-measure-theoretic setting. The gossip algorithm has been used to propagate local decisions of target detection over a mobile sensor network and thus, it generates a *belief* for the target detected over the network. The proposed concept has been validated through numerical experiments with a mobile sensor network and a point mass robot.**

1. Introduction

Autonomous robots are becoming ubiquitous and are envisaged to play an increasingly important role in both civilian and military applications. While operating in unknown and unstructured environments, they often have limited or unreliable long-range communication and GPS capabilities due to constraints on energy requirements and (or) adversarial environment. However, with the recent advances in sensing and low-complexity signal processing algorithms, sensors can locally detect regions of interest with high degree of accuracy, which reduces the communication overhead significantly. But it limits event awareness which limits performance in the network capacity. Consider, for example, a search and rescue operation which requires sequential collaboration between the '*search agents*' (agents with limited sensing abilities) and the '*rescue agents*' (agents capable of rescuing any sensed target in a region of interest). The efficacy of such missions in the network capacity would depend on how quickly the network of '*search agents*' can react to the sensed targets of interest and guide a '*rescue agent*' to the target of interest under the constraints of limited communication and global positioning. The goal of this paper is to present a framework for navigation of an autonomous agent in an unknown and GPS-denied environment with the help of a (possibly mobile) sensor network which serves the two-fold purpose of target localization and generating the way-points for the autonomous robot.

A lot of work has recently been reported on source seeking in sensor fields [1] [2] [3] [4]. The objective of such problems is to determine the minimum of an unknown signal field (the possible location of the source) using a stochastic gradient descent algorithm. Authors in [2] [4] present a multi-agent coordination framework for estimation of the peaks of sensor field. However, the agents have to communicate their sensor measurements and an artificial potential function is required to estimate the gradient of the sensor field. Several attempts have been made in literature to make use of static sensor networks to guide a robot [5], [6]. In [6], a psuedo-gradient calculated based on sensor readings in a static sensor network is used for localization and directed navigation of an autonomous robot in unknown environments. However, the algorithm presented can't be trivially extended to navigation using a mobile sensor network. Mobile sensor networks have potential advantage over their static counterparts in terms of coverage and time-criticality [7].

The current work builds on a recent work on distributed decision propagation in mobile ad-hoc sensor networks presented in [8] where the proximity network of an agent is modeled as a probabilistic finite state automaton (PFSA). An '*agent measure function*' is then defined (based on the recently reported language measure theory [9] [10]) for all the agents in the network which signifies its '*level of awareness*' regarding a locally sensed '*target*' in the operational area. In the current work, the *agent measure function* generated by a mobile sensor network is used to guide an autonomous robot through an unknown and unstructured environment. The current framework has the following potential advantages over those reported in literature:

- The algorithm only requires exchange of local decisions about sensed targets, and not the actual sensor measurements. This has the potential to significantly reduce communication overhead and makes the network more robust to communication flips.
- No artificial potential function is required to guide an agent to the locally detected goal; a gradient is automatically generated by the *agent measure function* which is maximized at the location of sensed target.

This is different from other works present in current literature [5] in the sense that not all sensors in the network detect the target; an awareness about the presence of a local target is developed via gossip and it is fed back to a continuous time controller of a robot to find a path. A sampling-based algorithm is used to tackle the dynamics of the robot at a lower continuous control level.

It is noted that the term *agent* is often interchangeably used for mobile sensor and shouldn't be confused for robot.

2. Background on Distributed decision propagation in Proximity networks

This section briefly summarizes the concept of real measure of probabilistic regular languages generated by a PFSA [10] [9] followed by the formulation of generalized gossip algorithm presented in [8].

A. Basic Notions of Language-Measure Theory

For brevity, the concept of real measures have been restricted to irreducible Markov Chain. Interested reader is referred to [9] [10] for further details.

Definition 2.1 *(Real Measure of Irreducible Markov Chain) Let a stationary Markov chain be denoted by the three-tuple $(Q, \mathbf{\Pi}, \boldsymbol{\chi})$, where Q is the set of states; the state transition function $\mathbf{\Pi} : Q \times Q \rightarrow [0, 1]$ represents the $|Q| \times |Q|$ stochastic matrix for the Markov chain ($|Q|$ represents the cardinality of the set of states); and $\boldsymbol{\chi} : Q \rightarrow \mathbb{R}$ is the vector-valued characteristic function that assigns a signed real weight to each state. A real measure $\nu_i(\theta)$ for state i is then defined as*

$$\nu_i(\theta) \triangleq \sum_{k=0}^{\infty} \theta(1-\theta)^k \mathbf{\Delta}_i \mathbf{\Pi}^k \boldsymbol{\chi}, \quad i = 1, 2, \cdots, |Q| \quad (1)$$

where $\theta \in (0, 1)$ is a user specified parameter and $\mathbf{\Delta}_i$ is defined as a $1 \times |Q|$ vector $[\delta_{i1}, \delta_{i2} \ldots \delta_{i|Q|}]$ which is given as δ_{ij}=1, if i=j, else zero. The expression for the measure in Equation 1 can be expressed as : $\boldsymbol{\nu}(\theta) = \theta(\mathbb{I} - (1 - \theta)\mathbf{\Pi})^{-1}\boldsymbol{\chi}$ The inverse is guranteed to exist for $\theta \in (0, 1)$. $\mathbf{\Delta}_i \mathbf{\Pi}^n$ represents the state probability vector at an instant n time-steps in the future for a Markov process beginning in state i and the expected value of the characteristic function is given by $\mathbf{\Delta}_i \mathbf{\Pi}^n \boldsymbol{\chi}$.

B. Background of Distributed Decision Propagation

This subsection briefly describes the formulation of the generalized gossip policy in the context of proximity networks proposed in [8]. Proximity network [11] is a particular formulation of time-varying mobile-agent networks, inspired from social networks where only proximal agents communicate at any given time epoch [12].

In the present context, proximal agents exchange information related to their beliefs regarding the environment. After the expiry of a message lifetime L_m, agents possibly update their beliefs based on their own observation and messages from other agents. There are two time-scales involved in this problem setup. In contrast to the faster time-scale (t) of agent motion, the algorithm for updating the agents' beliefs runs on a (possibly) slower time-scale (denoted by τ). The time-scale for updating the belief is chosen to be slower as it allows for sufficient interactions among the agents, especially if the density of agents is low. To capture temporal effects in a realistic setting, L_m should be appropriately chosen based on other network parameters.

With this setup, let a time-dependent (in the slow-scale τ) graph be denoted as G and a few related terms are defined as follows.

Definition 2.2 *(Adjacency Matrix [13]) The adjacency matrix A of the graph G is defined such that its element a_{ij} is unity if the agent i communicates with the agent j in the time period of L_m, else zero. To eliminate self-loops, each diagonal element of the adjacency matrix is constrained to be zero.*

Definition 2.3 *(Laplacian Matrix [13]) The Laplacian matrix ($\mathcal{L}$) of a graph G is defined as: $\mathcal{L} = D - A$ where the degree matrix D is a diagonal matrix with d^i denoting the degree of node i.*

Definition 2.4 *(Interaction Matrix [13]) The agent interaction matrix Π is defined as: $\Pi = I - \beta\mathcal{L}$*

The generalized gossip strategy involves two characteristic variables associated with each agent, namely the *state characteristic function* χ and the *agent measure function* ν. $\chi \in \{0, 1\}$ signifies whether an agent has detected a *target* ($\chi = 1$) or not ($\chi = 0$). $\nu \in [0, 1]$ signifies the level of awareness of an agent regarding the presence of a *target* in the surveillance region. It is noted that, Π, ν and χ are functions of the slow time-scale τ. In the above setting, a decentralized strategy for measure updating in the mobile-agent population is introduced below in terms of a user-defined control parameter $\theta \in (0, 1]$.

$$\nu_i(\theta)|_{\tau+1} = (1-\theta) \sum_{j \in \{i\} \cup Nb(i)} \Pi_{ij}|_\tau \; \nu_j(\theta)|_\tau + \theta\chi_i|_\tau \quad (2)$$

where $Nb(i)$ denotes the set of agents that communicate with the agent i during the time span between τ and $\tau + 1$. It is noted that while computing the future (awareness or belief) measure of an agent, the parameter θ controls the trade-off between the effects of current self-observation and current measures of all agents. In the vector notation, the dynamics can be expressed as: $\boldsymbol{\nu}(\theta)|_{\tau+1} = (1-\theta)\mathbf{\Pi}|_\tau \boldsymbol{\nu}(\theta)|_\tau + \theta\boldsymbol{\chi}|_\tau$. Thus, this policy is simply a gossip algorithm with varying input $\chi|_\tau$ and varying network topology represented by $\Pi|_\tau$. The memory of a past input fades as a function of the parameter θ. Due to this notion, the above policy can be called a *generalized gossip algorithm* with θ as the generalizing parameter.

In the following sections, agent measure function is often referred to as belief.

3. Problem Formulation

This section formulates the problem of path planning for an autonomous robot in the absence of GPS. For simplicity of exposition, we make certain assumptions to unambiguously present the efficacy of the proposed framework for reactive navigation in the absence of GPS, which are outlined below.

1) An autonomous robot can locally estimate relative positions of mobile sensors using state-of-art positioning techniques in sensor networks [14].
2) Mobile sensors and the autonomous robot are locally able to coordinate for collision avoidance.
3) Communication of the robot with other mobile sensors is considered in the time scale $\mathrm{T} >> \tau$.

Under these major assumptions, we consider the case of a set of mobile sensors performing surveillance in a region, where the task is to detect targets in a given region. For simplicity, the target (i.e., the goal for the autonomous robot) is modeled as a local region of interest in the surveillance region such that only a few sensors that search areas within the region of interest have a non-zero probability of detecting it. For clarity, a simplistic model for target detection is followed which is described next. A region of interest is modeled as a map for probability of detection of a target. Let the probability of detection of a target be denoted by P_D, which attains the maximum at the center of the target's physical location and decays to zero linearly with distance from the center in a radially symmetric manner. A region of interest is then, characterized by the following parameters:

- The maximum probability of detection of the target, P_{Dmax}
- The effective radius (r_{hs}) of the circular region within which $P_D > 0.5$,

In [8], a distributed decision propagation algorithm has been proposed for dissemination of the sensed target throughout the mobile sensor network. The aim of the current work is to develop a distributed navigation algorithm to help guide an autonomous robot to the detected region of interest (i.e., goal for the autonomous robot) where no one is aware of the sensed location of target and there is no GPS. Since the robot has only a finite sensing and communication radius, it can only be aware of the local belief in the network. The problem of reactive navigation to a locally detected target is then reduced to the recursive estimation of a sequence of way-points which the robot can follow to finally reach the goal. Even though the broad scope of this research is to allow a robot with any complicated dynamics find a feasible trajectory, for simplicity, this paper is limited to a point mass robot.

4. Proposed Approach

Under the framework presented in [8] and briefly explained in section 2, we present an algorithm which guarantees a unique maxima and a gradient towards the same in the proximity network. The idea is that if the autonomous robot moves in a way so that its belief (based on the belief of its nearby mobile sensors) monotonically improves (or increases) with movement, then under the condition that the belief of the network is maximized at the physical location of the goal, the robot will eventually reach the goal. Under the constraints of limited communication and sensing horizon, the robot has access to belief of only the local mobile sensors. However, due to the presence of a gradient towards the goal, the robot is able to estimate a waypoint where the belief is greater than its current belief. To this end, the robot learns an implicit correspondence between a geographical location and the belief in the network by using a multiple regression framework. The maximum of the implicit surface is the waypoint the robot moves to, over a certain time horizon till the next communication with the network is established. This is achieved by following a feasible trajectory obtained by sampling from the local configuration space using RRTs.These steps are recursively followed till the robot reaches the sensed region of interest i.e., the goal. The idea is similar to the commonly studied receding horizon motion planning framework, where a reactive plan is followed by the robot over a finite time horizon as a reaction to real-time information.

A. *Decentralized Gossip for Decision Propagation*

Based on the framework of generalized gossip algorithm, this subsection presents an algorithm which creates a bias in the proximity network of the mobile sensors towards the sensed region of interest The idea is based on optimal control theory of a PFSA [10] [15]. Under this umbrella, the belief of every sensor is maximized by averaging only over the set of neighbors that have belief greater than the sensor. In the original gossip strategy (see equation 2), a sensing agent is influenced by all its neighbors. However, to maximize its measure, an agent can follow a strategy where it is only influenced by neighbors that have a higher belief than its own belief This strategy is succintly presented in Algorithm 1. The key point is that the elements of the interaction matrix corresponding to agents with a lower measure are made zero. However, to keep the interaction matrix stochastic, those elements are adjusted as a self-loop to the agent (see steps 5 through 11 in Algorithm 1). Based on the results in [10] [15], this strategy ensures a maximum in the belief network at the goal region for the autonomous agent and at the same time, it creates a gradient towards the same. This biased approach ensures that a mobile sensor which is closer to the sensed region of interest will have a higher belief as compared to those further away from it.

B. *Implicit Surface based Interpolation for Navigation*

Under the assumption that the robot can localize mobile sensors in its neighborhood [14], beliefs of the mobile sensors in the robot's neighborhood are used to learn an implicit correspondence between a physical location and belief.

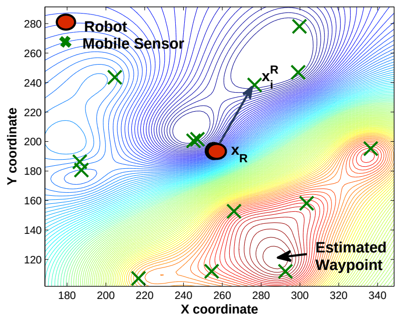


Fig. 1. The robot can use relative locations of sensors to estimate a local implicit correspondence between a physical location in its neighborhood and the measure function ν. Note that this figure shows an example scenario.

In this regard, let $x_R \in \mathbb{R}^2$ be the location of the robot at some time instant $\mathrm{T}_i, i \in \mathbb{N}$ and $x_S \in \mathbb{R}^2$ be the location of the target detected by the mobile sensor network. Let Nb(R) be the local neighborhood of the robot in which it can locally estimate the positions of mobile sensors within its communication range. Let x^R denote the relative coordinate of a physical location measured w.r.t. the robot in the region Nb(R). In this setting, let $x_1^R, x_2^R, \ldots, x_M^R$ be the relative positions of the mobile sensor w.r.t. the robot . We assume that the robot can accurately (or with some bounded uncertainty) estimate x_i^R, $i = 1, 2, \ldots, M$, using localization techniques in sensor network. The interpolation problem is then, formally stated as follows: Given the approximate locations of the neighbors of the robot, $\{x_i^R \in \mathbb{R}^2, i = 1 \text{ to M}\}$ and their corresponding beliefs $\{\ \nu_i \in \mathbb{R}\ \}$, a function $\mathfrak{F} : \mathbb{R}^2 \rightarrow \mathbb{R}$ is estimated, such that it satisfies the boundary constraints

$$\mathfrak{F}(x_i^R) = \nu_i, i = 1, 2, \ldots, M \tag{3}$$

where $\mathfrak{F}(x^R)$ has the following form

$$\mathfrak{F}(x^R) = \sum_{i=1}^{M} w_i \phi(||x^R - x_i^R||) \tag{4}$$

w_i are weights assigned to the individual RBF's. It can be shown that any continuous function on a compact interval can, in principle, be interpolated with arbitrary accuracy by a sum of the form(4), if a sufficiently large number of RBFs are used [16]. The function $\mathfrak{F}$ represents an implicit correspondence between the local physical locations and belief about region of interest.

Under this setting,

$$x_{max}^R = \arg \max_{x^R \in N\text{b(R)}} \mathfrak{F}(x^R) \tag{5}$$

and,

$$\nu_{max} = \max_{x^R \in N\text{b(R)}} \mathfrak{F}(x^R) \tag{6}$$

$x_{max}^R(\mathrm{T}_i)$ is then, the estimated waypoint to which the robot needs to move, over the next time horizon $(\mathrm{T}_i, \mathrm{T}_{i+1}]$. Let us assume that $\{x_{max}^R(\mathrm{T}_1), x_{max}^R(\mathrm{T}_2), \ldots, x_{max}^R(\mathrm{T}_n)\}$ is the sequence of way-points estimated by the robot in the slow time scale at instants $\mathrm{T}_1, \mathrm{T}_2, \ldots, \mathrm{T}_n$. Then, if the robot moves in a way such that $\{\nu_{max}(\mathrm{T}_1), \nu_{max}(\mathrm{T}_2), \ldots, \nu_{max}(\mathrm{T}_n)\}$ is a monotonically increasing set, then the following will hold:

$$\text{dist}\left(\lim_{n \to \infty} x_{max}^R(\mathrm{T}_n), x_S\right) < \epsilon \tag{7}$$

In this work, an inverse multi-quadric form of ϕ was chosen. Specifically, $\phi(||x^R - x_i^R||)$ is chosen to be, $1/\sqrt{r^2 + \sigma^2}$, where r = $||x^R - x_i^R||_2$ and σ is chosen as k/θ. As it was explained before in section 2, θ is the generalizing parameter for the gossip algorithm. In order to determine $\{w_i, i = 1 \ \text{ to } \ M\}$, a multiple regression algorithm is used. The procedure is succinctly presented in Algorithm 2.

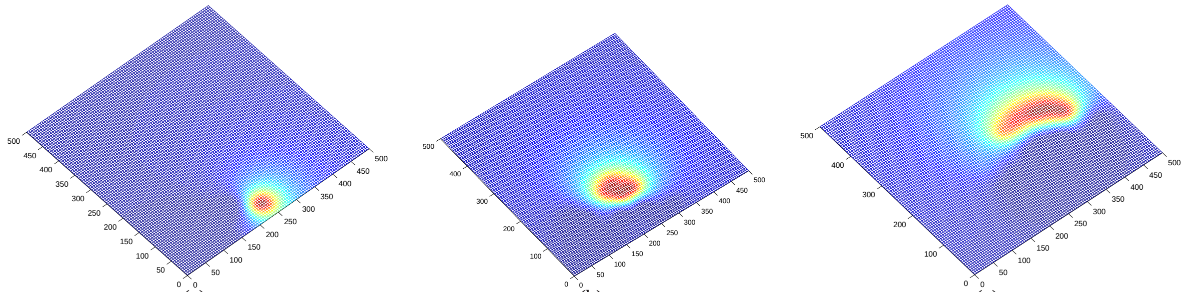
C. RRT based Navigation

In the last step, the robot gets an estimate of the waypoint it should move to. Based on the current location and the waypoint found in the last step, a rapidly exploring random tree (RRT) is built in an anytime fashion to find a feasible trajectory for the robot. Under this framework, we assume that the robot can avoid the static obstacles by building collision-free trees [17] For completeness of the paper, RRT has been succintly explained in Algorithm 3.

Algorithm 1 Distributed belief updating strategy for mobile sensors

while true do
  for all sensors 'i' in the network **do**
    if $Nb(i) \neq 0$ **then**
      $d_i = \text{CARD}(Nb(i))$
      $/ ** $ Begin Infinite Asynchronous loop $** /$
      $/ **$ Query $\nu(\theta)_j \ ** /$
      if $\nu_j(\theta)|_\tau \leq \nu_i(\theta)|_\tau$ **then**
        $\Pi_{ii}|_\tau = \Pi_{ii}|_\tau + \Pi_{ij}|_\tau$
        $\Pi_{ij}|_\tau = 0$
      end if
      if $\nu_j(\theta)|_\tau > \nu_i(\theta)|_\tau$ & $\Pi_{ij}|_\tau = 0$ **then**
        $\Pi_{ij}|_\tau = 1/d_i$
        $\Pi_{ii}|_\tau = \Pi_{ii}|_\tau - 1/d_i$
      end if
    end if
    $\nu_i(\theta)|_\tau = (1-\theta)\sum_{j \in \{i\} \cup Nb(i)} \Pi_{ij}|_\tau \nu_j(\theta)|_\tau + \theta\chi^i|_\tau$
  end for
end while

Remark 4.1 ***Correctness***: *The plan will always give the robot a path to the sensed goal. This is argued by making some observations. Due to the biased gossip algorithm based on the optimal control of a weighted PFSA, it is ensured that there is a gradient towards the goal. Under the assumption of*



(a) (b) (c)

Fig. 2. The implicit surface estimated by the robot at three different time instants in its local neighborhood during navigation to an unknown goal location. The goal is at [450,250].

Algorithm 2 Navigation of the robot

while dist(x_R, x_S)$>\epsilon$ **do**
  Solve $\mathfrak{F}(x^R) = \sum_{i=1}^{M} w_i \phi(||x^R - x_i^R||)$ using boundary constraints $\{x_i^R, \nu_i\}, x_i^R \in N\text{b(R)}$
  Use $\mathfrak{F}(x^R)$ to estimate $x^R_{max} = \arg\max_{x^R \in N\text{b(R)}} \mathfrak{F}(x^R)$ and $\nu_{max} = \max_{x^R \in N\text{b(R)}} \mathfrak{F}(x^R)$
  RRT$(x_R, K, \Delta t, x^R_{max})$
  $/**$ For the function RRT, see Algorithm 3 $**/$
end while

Algorithm 3 RRT

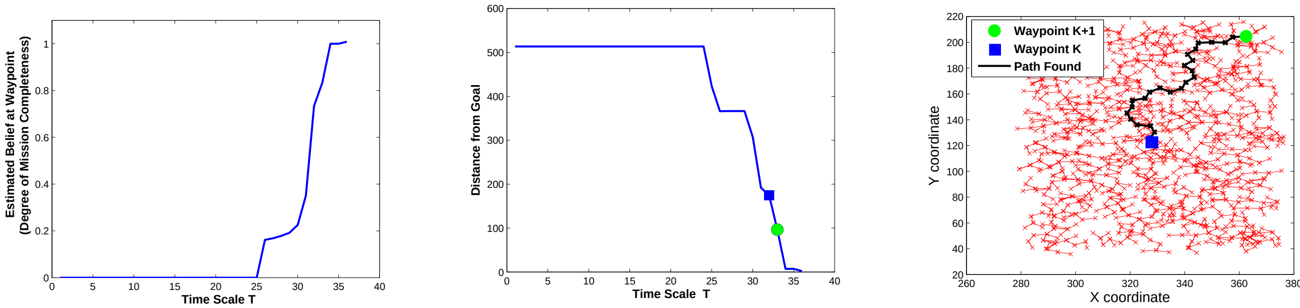
Input:$(q_{init}, K, \Delta t, q_{goal})$
Output: Tree G with a path P from q_{init} to q_{goal}
for k = 1 to K **do**
  $q_{rand} \leftarrow$ RandConf()
  $q_{near} \leftarrow$ NearestVertex(q_{rand}, G)
  $q_{new} \leftarrow$ NewConf$(q_{near}, q_{rand}, \Delta t)$
  Add vertex q_{new} to G
  Add edge (q_{near}, q_{new}) to G
end for
$qg_{near} \leftarrow$ NearestVertex(q_{goal}, G)
Retrace a path P from qg_{near} to q_{init} over G.
return P

bounded uncertainties in the localization estimates of sensors within its communication radius, the robot can always locate a way-point which has a higher belief (as found by the interpolation function) than its current belief. As the measure is maximized at the target location, so, as long as the robot moves in such a way that its measure (i.e., belief about the presence of a goal) monotonically increases, it will end up at the goal.

5. Results for an Example Problem

This section presents results of numerical experiments for an example problem of surveillance and reconnaissance which involves a mobile sensor network and an autonomous robot which needs to navigate to a target detected by the mobile network. We consider a surveillance example for a region of area A performed by N mobile sensors, where each mobile sensor has a communication radius R_s. The robot has a communication and sensing radius R_r. The individual mission of the agents is to detect any target and communicate this to their neighbors. The global mission objective of the sensor network is to direct a robot with greater capabilities to the sensed region with target for neutralization of threat or to deliver a service. For the simulation study, the parameters are chosen as: $A = 500^2, N = 150, R_s = 50,$ and $R_r = 100$. For modeling of target, the value of P_D was chosen to be 0.9 and r_{hs} was chosen to be 20. The generalized gossip parameter θ was chosen to be 0.02. The velocity of the mobile sensors in the network was chosen to be 5 and the maximum velocity for the robot was 10. The mobile sensors are moving in the region with a 2-D random walk fashion with the constant velocity. A slower velocity for the mobile sensors might result in a slower information propagation but, it results in more stable local dynamics for the robot. Target is located at $[450, 250]$ while the robot is at $[1, 1]$ to begin with. ϵ (see equation 7) is chosen to be equal to r_{hs}.

The robot starts moving towards the goal as soon as its local neighborhood becomes aware of the target detection through gossip. Once the robot becomes aware of the detection, it makes use of the disseminated distributed belief about the target to find a path to the target. Figure 2 shows the implicit surface for correspondence between the *agent measure function* and a geographical location in its local neighborhood at different time instants based on the communication with the mobile sensors in its communication radius. Communication is re-established after the robot reaches the estimated waypoint corresponding to the local implicit surface. Note that the different surfaces are estimated in the slow time scale T. Figure 3(a) shows the monotonic increment in the belief at the estimated waypoint based on the local implicit surface with the movement of the robot. It can be seen in Figure 3(a) that as soon as the robot becomes aware of a sensed target (belief > 0), it is able to move in such a fashion that its awareness about the presence of the target monotonically increases and finally converges to its maximum value as it reaches the goal. The belief of the way-points could also be used as a measure for degree of



(a) (b) (c)

Fig. 3. Plates (a) shows the monotonic improvement in belief of the estimated way-points in the slow time scale (T); It is also considered as the degree of completeness of the mission. Plate (b) shows the monotonic decrease in the euclidean norm between the robot's location and the goal, measured in the slow time scale T. Plate (c) shows the tree representing the actual trajectory of the robot between two consecutive way-points (see the blue square and green circle in plate (b)). Note that for clarity only a part of actual trajectory has been shown.

completion of the mission; convergence of the belief to the maximum value suggests mission completion. Figure 3(b) shows the monotonic convergence of the robot's movement to the unknown goal location under the proposed framework. Figure 3(b) shows the inherent goal-directedness in the robot's motion once it *sniffs* (i.e., belief > 0) the presence of the target Figure 3(b) shows Euclidean distance between the robot and the goal in the slow time scale T. Figure 3(c) shows the actual trajectory found by the lower level continuous controller and followed by the robot between two consecutive way-points located at $[327, 125]$ and $[360, 205]$ (see the blue square and green circle in plates (b) and (c). It is noted this is not the complete path as the initial location of the robot is at $[1, 1]$. For clarity of presentation, only a part of the entire path has been shown).

6. Conclusions and Future Work

The paper presents a framework of hierarchical planning, where the dynamic knowledge of the goal is propagated over a mobile sensor network, which is used as a feedback for a low-level continuous controller to find a feasible path for the robot. Using a controlled gossip algorithm and sequential estimation of the way-points locally, it is shown that the robot is capable of finding a path to the goal point. However, the efficacy of the proposed path planning algorithm is contingent on the accuracy of the localization techniques executed over the sensor network.

The following topics are recommended for future research.

1) Extension to planning in presence of multiple targets and multiple regions of interest.
2) Identification of an explicit relationship between imperfections in localization in the sensor network and the navigation of the robot.

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