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Hierarchical Optimization for Building Energy Systems

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***Abstract*— Optimal tradeoff between energy efficiency and thermal comfort is a critical aspect for building heating, ventilation, and air-conditioning (HVAC) systems. Traditional optimization and control schemes such as, PID and rule-based control (RBC), may not sufficiently address this issue in smart buildings. Moreover, most optimization-based previous works have only considered either water-side HVAC or air-side HVAC separately while both of these systems significantly affect the performance of each other. This paper presents a hierarchical optimization approach to take water-side and air-side HVAC systems into account simultaneously for energy efficiency and thermal comfort requirements. We establish an outer-inner loop algorithmic framework and develop the hierarchical gradient descent algorithm and its variants to search for optimal set points. A notion of communication period is also introduced to control the computational complexity of the algorithm. A numerical case study is used for demonstrating the efficacy of the proposed approach.**

1. Introduction

Smart buildings have received considerable attentions recently both from academia and industry and thus energy efficiency is one of correspondingly emerging topics in building heating, ventilation, and air-conditioning (HVAC) systems as well as grids. Conventional sensing and control techniques, such as PID [1] and rule-based control (RBC) [2] may not sufficiently satisfy requirements by such building energy systems and grids. In addition, past work has primarily focused on either water-side, air-side HVAC, or power systems separately [3], [4], [5], [6], [7]. However, there exists an inherent hierarchical structure between grid and buildings [8] such that developing hierarchical control and optimization approaches become essential.

This paper presents a hierarchical optimization framework by taking water-side and air-side subsystems into account. Specifically, we develop a hierarchical gradient descent approach to iteratively find out the optimal set point in water-side subsystem (i.e., chilled water temperature) and the optimal set point in air-side subsystem (i.e., supply air temperature), respectively. An outer-inner loop algorithmic framework is accordingly established and a communication period parameter is identified for controlling the computational complexity. In this context, while a simple numerical case study is used to validate the proposed approach, the extension of the hierarchical optimization framework can be made by incorporation of PID and RBC to satisfy different building configurations.

Related Work: The authors in [9] presented global optimization technologies for overall HVAC systems transforming and simplifying the original problem into a compact form. Three levels fuzzy controllers were developed for low-energy buildings and compared with a supervisory control strategy based on expert rules [10]. [11] presented a model-based hierarchical optimal control scheme for regulating air flow, comfort, and energy consumption. In [12], the authors proposed a hierarchical combined heat and power optimal control algorithm which had the potential for energy savings. In [13], the authors developed a hierarchical design optimization model for facilitating large-scale and simulation-based design tasks in architecture. Additional works on hierarchical control methods [14], [15], [16], and bi-level optimization based on hierarchical evolutionary algorithm [17] were proposed to find the good-quality control and optimization strategy.

2. Problem Formulation

This section states the problem formulation for the hierarchical optimization in building energy systems. Figure 1 shows the general layout for HVAC systems with air handling unit (AHU), zone, chilled water system, and boiler system.

Outside air and return air from the zone, whose air flows are controlled by dampers, is combined to produce mixed air. Mixed air passes through the cooling or heating coil to be cooled down or heated up by the chilled water system or boiler system based on requirements to generate supply air, which is pumped into the local zone. Before entering the local zone, supply air may be reheated by reheat coil in variable air volume (VAV). Return air from local zone is circulated back to AHU for next cycle.

In this study, only cooling mode is considered for the analysis. However, it is noted that heating mode follows similar procedures for analysis. Therefore, inlet chilled water temperature set point is the variable to be optimized in the outer loop of hierarchical optimization framework. Note, the dynamics of the chilled water system is not considered here for simplicity. Therefore, the actual chilled water temperature is the same as the chilled water temperature set point. For the inner loop, supply air temperature set point is the variable to be optimized. In this paper, energy consumption is primarily the energy consumed in the chilled water system and cooling coil. It should be noted that in VAV reheat energy can be taken into account. However, it is omitted in this paper for simplicity.

The generic hierarchical optimization problem typically involves two levels of optimization problems in which one optimization problem contains the other. These two levels of optimization problems have their own objective functions and constraints, which thus results in two classes of variables, i.e., the upper level [18] variables (denoted by $\mathbf{x} \in \mathcal{R}^n$) and the lower level (or follower level) variables (denoted by $\mathbf{y} \in \mathcal{R}^m$). Equivalently, the lower level optimization can be regarded as a parametric optimization problem which is solved with respect to $\mathbf{y}$ while $\mathbf{x}$ acts as parameters. On the other hand, the lower level optimization problem acts as a constraint to the upper level optimization problem. Therefore, it implies that the feasible solutions for the lower level optimization problem need to satisfy the upper level constraints.

Definition 2.1 *Given an upper level objective function $\mathcal{F} : \mathcal{R}^n \times \mathcal{R}^m \to \mathcal{R}$ and a lower level objective function $f : \mathcal{R}^n \times \mathcal{R}^m \to \mathcal{R}$, the hierarchical optimization problem is given by*

$$\begin{aligned}&\min_{\mathbf{x}\in\mathcal{X},\mathbf{y}\in\mathcal{Y}}\mathcal{F}(\mathbf{x},\mathbf{y})\\ &\text{s.t. } \mathcal{G}_l(\mathbf{x},\mathbf{y}) \le 0, l=1,\dots,L\\ &\mathbf{y}\in \text{argmin}_{\mathbf{y}\in\mathcal{Y}}\{f(\mathbf{x},\mathbf{y}) : g_q(\mathbf{x},\mathbf{y}) \le 0, q=1,\dots,M\}\end{aligned}$$

where $\mathcal{X} \subset \mathcal{R}^n$ is convex and compact, $\mathcal{Y} \subset \mathcal{R}^m$ is convex and compact, $\mathcal{G}_l : \mathcal{X} \times \mathcal{Y} \to \mathcal{R}, l = 1,\dots,L$ signifies the upper level constraints, $g_q : \mathcal{X} \times \mathcal{Y} \to \mathcal{R}, q = 1,\dots,M$ represents the lower level constraints.

We further simplify the problem formulation stated above using set-valued mapping. The equivalent expression can be as follows

Definition 2.2 *Give that $\Phi : \mathcal{R}^n \rightrightarrows \mathcal{R}^m$ be a set-valued mapping such that*

$$\Phi(\mathbf{x}) = \text{argmin}_{\mathbf{y}\in\mathcal{Y}}\{f(\mathbf{x},\mathbf{y}) : g_q(\mathbf{x},\mathbf{y}) \le 0, q=1,\dots,M\}$$

which indicates the constraint defined by the lower level optimization problem, namely, $\Phi(\mathbf{x}) \subset \mathcal{Y}$ for each $\mathbf{x} \in \mathcal{X}$. We have,

$$\begin{aligned}&\min_{\mathbf{x}\in\mathcal{X},\mathbf{y}\in\mathcal{Y}}\mathcal{F}(\mathbf{x},\mathbf{y})\\ &\text{s.t. } \mathcal{G}_l(\mathbf{x},\mathbf{y}) \le 0, l=1,\dots,L\\ &\mathbf{y}\in\Phi(\mathbf{x})\end{aligned}$$

The Definition 2 shows that Φ can be intuitively regarded as a parameterized range-cosntraint for the lower level variable $\mathbf{y}$.

Solving the hierarchical optimization problem in either Definition 2.1 or Definition 2 can be NP-hard [19] and state-of-the-art can be found in [20]. Note, that in this context the upper and lower level variables are vectors while they degenerate to scalars when defined for the HVAC system. We have shown the problem formulation while there is still a lack of clarity regarding which optimal solution should be adopted for solving the upper level optimization problem given multiple lower optimal solutions. In literature [21], two scenarios in which the leader in the upper level optimization problem defines two different positions, i.e., optimistic position and pessimistic position, respectively, are widely and well studied.

Optimistic Position: In this scenario, in the presence of multiple lower level optima, the leader takes its expectation on the follower to select a certain solution from the optimal lower level set $\Phi^*(\mathbf{x})$, which results in the optimal objective function value at the upper level. The optimal choice from the feasible set of the lower level optimization problem in this context can be defined as follows:

$$\Phi^*(\mathbf{x}) = \text{argmin}_{\mathbf{y}\in\mathcal{Y}}\{\mathcal{F}(\mathbf{x},\mathbf{y}) : \mathbf{y}\in\Phi(\mathbf{x})\} \tag{1}$$

Thus, the hierarchical optimization problem in an optimistic position scenario is defined below:

$$\begin{aligned}&\min_{\mathbf{x}\in\mathcal{X},\mathbf{y}\in\mathcal{Y}}\mathcal{F}(\mathbf{x},\mathbf{y})\\ &\text{s.t. } \mathcal{G}_l(\mathbf{x},\mathbf{y}) \le 0, l=1,\dots,L\\ &\mathbf{y}\in\Phi^*(\mathbf{x})\end{aligned}$$

Pessimistic Position: In a pessimistic position, the leader in upper level optimization problem chooses the worst lower level optimal solution from the optimal set which leads to the worst objective function value at the upper level. Hence, we similarly define such a worst case choice function as follows:

$$\Phi^p(\mathbf{x}) = \text{argmax}_{\mathbf{y}\in\mathcal{Y}}\{\mathcal{F}(\mathbf{x},\mathbf{y}) : \mathbf{y}\in\Phi(\mathbf{x})\} \tag{2}$$

The hierarchical optimization framework in a pessimistic position can be cast below:

$$\begin{aligned}&\min_{\mathbf{x}\in\mathcal{X},\mathbf{y}\in\mathcal{Y}}\mathcal{F}(\mathbf{x},\mathbf{y})\\ &\text{s.t. } \mathcal{G}_l(\mathbf{x},\mathbf{y}) \le 0, l=1,\dots,L\\ &\mathbf{y}\in\Phi^p(\mathbf{x})\end{aligned}$$

Typically, when hierarchical optimization problem is in an optimistic position, with a strictly convex lower level problem, it degenerates to a single level using the variational inequality. However, for a scenario with a pessimistic position, such a single level reduction is essentially intractable. Hence, we only consider the implementable optimistic position case in this paper.

Assumption 2.1 *a) For any $\mathbf{y} \in \mathcal{Y}$, $\mathcal{F}$ is Lipschitz continuous with respect to (w.r.t) $\mathbf{x} \in \mathcal{X}$ and is sufficiently smooth; b) for any $\mathbf{x} \in \mathcal{X}$, f is strictly convex w.r.t $\mathbf{y} \in \mathcal{Y}$ and is sufficiently smooth; c) for any l and q, $\mathcal{G}_l$ and g_q are sufficiently smooth; d) constraint set Φ is non-empty and compact.*

In the main result, a constraint qualification called Mangasarian-Fromovitz constraint qualification [22] is used

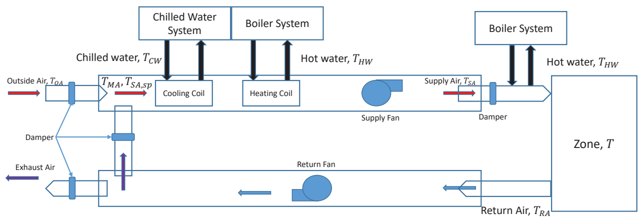


Fig. 1. *General layout of HVAC systems in buildings*

such that we introduce the definition as follows for completeness.

Definition 2.3 *Given an optimization problem in the form*

$$\min_{\mathbf{x}\in X} f(\mathbf{x}) \tag{3}$$

in which

$$\begin{aligned} X =&\{\mathbf{x}\in\mathcal{R}^n | g_u(\mathbf{x})\le 0, h_v(\mathbf{x})=0;\\ &u=1,\dots,U, v=1,\dots,V\}\end{aligned} \tag{4}$$

and all functions are continuously differentiable. Then a valid point $\hat{\mathbf{x}}$ is with ***Mangasarian-Fromovitz constraint qualification*** *if the two following conditions are satisfied: 1) the gradients of the equilibrium conditions $h_v(\mathbf{x})$ are linearly independent at the point $\hat{\mathbf{x}}$; 2) there is a vector $d\in\mathcal{R}^n$, such that $\nabla h_v(\hat{\mathbf{x}})^T d = 0$ and $\nabla g_u(\hat{\mathbf{x}})^T d<0$, if $g_u(\hat{\mathbf{x}})=0$.*

We are now ready to state the main result for the hierarchical optimization in an optimistic position.

Proposition 2.1 *Let Assumption 2.1 hold. If the Mangasarian-Fromowitz constraint qualification holds at all points, then the hierarchial optimization problem with the optimistic position is guaranteed to have optimal solutions.*

Convergence to optimal points was discussed using bi-level direct search method [23] and bi-level stochastic gradient method [20], respectively. In this paper, a new algorithmic framework with a user-defined communication period is proposed in the next section. In this context, the hierarchical optimization problem associated with HVAC systems can be obtained as follows

$$\begin{aligned} &\min_{T_{CW}\in\mathcal{X}, T_{SA}\in\mathcal{Y}} \mathcal{F}(T_{CW}, T_{SA})\\ &\text{s.t. } \mathcal{G}_l(T_{CW}, T_{SA})\le 0, l=1,\dots,L\\ &T_{SA}\in\Phi^*(T_{CW})\end{aligned}$$

It is noted that, the dimension of $\mathcal{X}$ and $\mathcal{Y}$ is 1. In this context, constraints can be box constraints due to capacities of actuators and zone comfort requirements in the HVAC system.

3. Methodology and Framework

This section presents the proposed scheme using hierarchical projected gradient descent with a communication period to control the computational complexity. The procedure for solving the problem is described briefly as follows: 1) the leader on upper level makes the decisions; 2) the leader asks followers on lower level to calculate their optima independently (in this study the number of followers is 1) if communication is "activated"; 3) followers on lower level send their decisions to the leader; 4) the leader modifies decisions obtained with consideration of overall benefit (minimization of energy consumption).

The specific algorithmic framework is introduced in this context.

Algorithm 1: Hierarchical Projected Gradient Descent (HPGD)

```
Initialization: x_0, y_0, α, τ
k, j ← 0
if (stopping criteria (σ) not satisfied) then
    Calculate gradient ∇_{x_k} F
    Projection: x_{k+1} ← P_X(x_k − α∇_{x_k} F)
    if mod(τ, k) = 0 then
        if (stopping criteria (ε) not satisfied) then
            Calculate gradient ∇_{y_j} f
            Projection: y_{j+1} ← P_Y(y_j − α∇_{y_j} f)
        end
        j ← j + 1
    end
    k ← k + 1
end
```

Remark 3.1 *From the algorithmic framework, it can be observed that a user-defined parameter τ, i.e., communication period, is used for controlling the frequency of solving the lower level problem (inner loop). Such a setup is capable of lowering the computational complexity. For example, in the HVAC system, if one needs $\mathcal{O}(p)$ to find out the optimal chilled water temperature, without communication period, the computational complexity may be $\mathcal{O}(p\times q)$ given that the computational complexity of finding the optimal supply air temperature is $\mathcal{O}(q)$. Introduction of the communication period can reduce the computational complexity to $\mathcal{O}(\frac{p\times q}{\tau})$.*

The momentum variant and averaging gradient variant of hierarchical projected gradient descent algorithm as well as the mix of them. The algorithmic frameworks of momentum variants are omitted in this paper due to the space limits.

Specifically, in the HVAC system, we have the following algorithm flow chart shown in Fig. 2. The proposed hierarchical projected gradient descent algorithm is applied to find out the optimal chilled water temperature set point and supply air temperature set point. Objective functions are energy consumption, which, however, may not be convex in practice. In this paper, on the purpose of validating the proposed approach, we use quadratic energy consumption function that is assumed to be strictly convex and continuously differentiable.

Algorithm 2: Hierarchical Projected Averaging Gradient Descent (HPAGD)

```
Initialization: x_0, y_0, α, τ
k, j ← 0
if (stopping criteria (σ) not satisfied) then
    Averaging gradient Δ_k = 1/(k+1) Σ_{i=0}^{k} ∇_{x_k} F
    Projection: x_{k+1} ← P_X(x_k − αΔ_k)
    if mod(τ, k) = 0 then
        if (stopping criteria (ε) not satisfied) then
            Averaging gradient Δ_j = 1/(j+1) Σ_{i=0}^{j} ∇_{y_j} f
            Projection: y_{j+1} ← P_Y(y_j − αΔ_j)
        end
        j ← j + 1
    end
    k ← k + 1
end
```

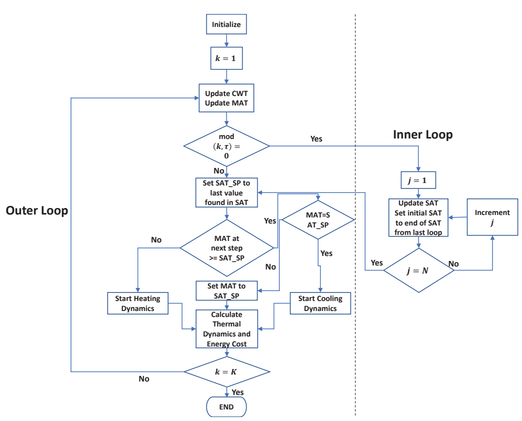


Fig. 2. *Algorithm flow chart of HVAC system using hierarchical projected gradient descent: CWT: chilled water temperature; HWT: hot water temperature; MAT: mixed air temperature; SAT_SP: supply air temperature set point; SAT: supply air temperature; K: the number of iterations of outer loop; N: the number of iterations of inner loop; τ: communication period.*

The cooling and heating dynamics describes the relation between mixed air temperature and supply air temperature, which is affected accordingly by the chilled water temperature or hot water temperature. For simplicity, we omit the reheat dynamics while using a linear function to describe the relation between supply air temperature and discharge air temperature in zone thermal dynamics for this study. As shown in the figure, communication period is used to "activate" the inner loop, which can reduce the computational complexity. Now we state the algorithm flow in detail.

After appropriate initializations, the outer loop is implemented subsequently by first updating the chilled water temperature, hot water temperature, and mixed air temperature. Then, whenever the communication period divides the number of iterations (this is checked with the modulo operator) the inner loop is carried out. Inside the inner loop, supply air temperature is updated with the initialization being the end value from the last loop. Inside the outer loop, the optimal supply air temperature obtained from the inner loop is used as the supply air temperature set point, which is used to activate the controller to control valve positions. From the figure, the cooling or heating coil cools down or heats up mixed air to become supply air based on the difference between mixed air temperature and supply air temperature set point. Therefore, the supply air temperature is used for calculating thermal dynamics and energy cost. The outer loop is repeatedly implemented before the number of iterations exceeds the total K.

Mathematically, the cooling dynamics for supply air temperature is a function of mixed air temperature, chilled water temperature, mixed air mass flow rate, and chilled water mass flow rate. Formally, it can be described as follows

$$T_{SA} = G(T_{MA}, T_{CW}, \dot{m}_{MA}, \dot{m}_{CW}) \quad (5)$$

Typically, G is represented by a nonlinear differential equation and solving such an equation is essentially intractable. For convenience, we do not directly solve it, but rather linearize and convert it into a transfer function based on which a control block diagram is established for the cooling dynamics.

We next discuss the thermal dynamics. It can be formally expressed by the following formula

$$T = H(T_{SA}, \dot{m}_{SA}) \quad (6)$$

As mentioned above, in this study the thermal dynamics for zone is simplified such that we use a simple linear function to indicate discharge air temperature. To avoid applying a physical model, we also use an autoregressive model in thermal dynamics which results in the following equation:

$$T(k+1) = H(T(k), T_{SA}(k), \dot{m}_{SA}(k)) \quad (7)$$

In this paper, energy cost is primarily considered as energy consumption in the chilled water system and the cooling coil in AHU. Due to the simplified reheat dynamics in VAV, reheat energy is omitted accordingly. We assume that energy cost is of quadratic form for validation of the proposed schemes.

4. Numerical Case Study

This section presents a numerical case study based on an HVAC system involving chilled water system, boiler system, AHU, and local zone. In this case the upper level variable is chilled water temperature (for simplicity, we do not consider hot water temperature optimization while in simulation it is fixed if using heating dynamics) and the lower level variable is supply air temperature. White noise processes are incorporated into gradient updates.

Results and Discussion: The upper level and lower level objective functions are energy cost in chilled water system and cooling coil in AHU, respectively. Moreover, in this case, the thermal comfort is required by maintaining zone temperature in between 69°F and 74°F. Figure 3 shows the zone temperature evolution during optimization in which the initial zone temperature is 63°F and the thermal comfort requirement is maintained as the stable zone temperature is approximately 73°F. Figure 4 depicts the energy cost which reduces along iterations using five different methods, i.e., HPGD, M-HPGD (the momentum variant of HPGD), HPAGD, and M-HPAGD (the momentum variant of HPAGD). We also consider M-HPGD with dynamic momentum term. It is noted that the momentum term constant is set 0.95. In these five approaches, M-HPGD outperforms the other four in terms of convergence rate, which suggests that the momentum term involving the previous step information speeds up the convergence of the proposed algorithm. While HPAGD is able to improve the convergence speed compared to HPGD, along iterations the accuracy may be worse than that of HPGD since it can be observed that the curve of HPAGD is not monotonically decreasing. M-HPAGD has better convergence rate than both HPGD and HPAGD, and the curve oscillates with smaller variance. Another observation that can be made is that M-HPGD with dynamic momentum term performs quite similarly compared to M-HPGD with constant momentum term, but outperforms the other three schemes. In summary, the proposed scheme is effective in reducing energy consumption for hierarchical structure of the building HVAC system.

Figures 5 - 7 show the evolution of chilled water temperature set point and supply air temperature set point along iterations using different approaches with different communication periods. The optimal chilled water temperature set point in our case is 43°F while the optimal supply air temperature set point is 74°F. Unsurprisingly, for the chilled water temperature set point, its evolution has a similar trend as objective function value. However, the effect of communication period is small since it only controls the iterations of the lower level variable, i.e., the supply air temperature set point. Moreover, averaging gradient can to some extent speed up the convergence to the optimal chilled water temperature set point at the beginning while it eventually causes a large deviation from the optimal solution. Such a phenomenon can be observed from the curves of HPAGD and M-HPAGD. In addition, dynamic momentum term may introduce variance to the convergence of the upper level optimizer from the comparison between M-HPGD and M-HPAGD.

We now analyze the properties of lower level optimizer. For the supply air temperature set point, the communication period has a significant impact on it. By visualizing the difference when τ is diverse (10, 50, and 500 respectively in Figures 5 - 7), it can be concluded that with a larger communication period, the supply air temperature set point can be constant for a longer time, as shown in Figure 7. This may be helpful for maintaining the stability of building HVAC systems. Frequently varying the supply air temperature set point may result in a highly time-varying supply air temperature in the HVAC system such that the valve associated with the heating or cooling coil should frequently change its position under the controller. This may damage some physical actuators correspondingly.

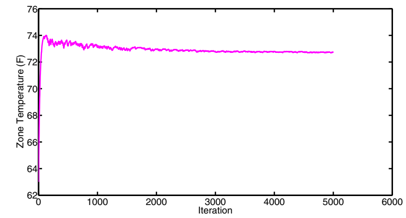


Fig. 3. *Zone temperature evolution using HPGD*

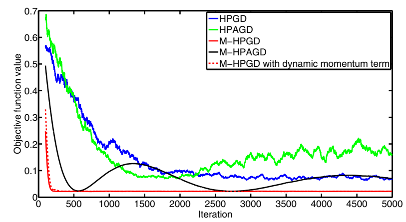
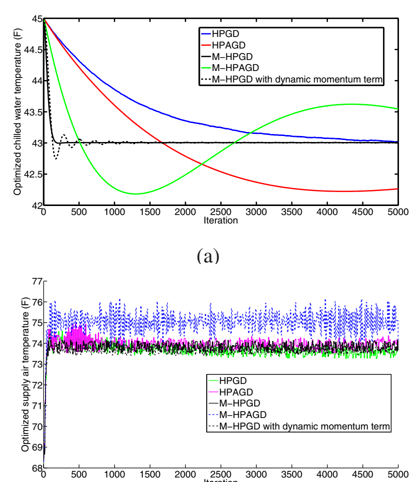


Fig. 4. *Energy cost using different schemes with $\tau = 10$; dynamic momentum term is $\frac{k}{k+3}$, where k is the number of iterations*

5. Conclusions and Future Works

This paper presents a hierarchical optimization framework for building HVAC system using hierarchical projected gradient descent approach and a user-defined communication period. Such a framework has been shown to be applicable in an HVAC system involving chilled water system, boiler system, AHU, and zone. A numerical case study is used to validate the proposed algorithms and it shows the energy con-



(a)

(b)

Fig. 5. *Optimized (a) chilled water temperature set point and (b) supply air temperature set point using different methods with $\tau = 10$; dynamic momentum term is $\frac{k}{k+3}$, where k is the number of iterations*

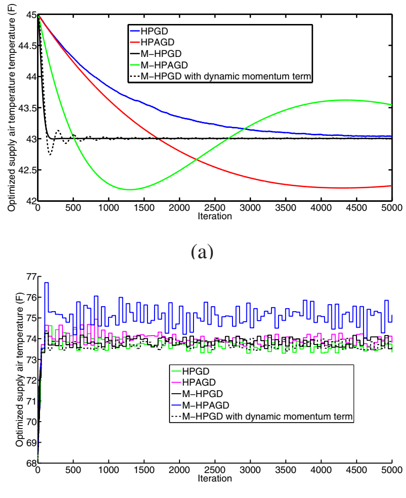


Fig. 6. *Optimized (a) chilled water temperature set point and (b) supply air temperature set point using different methods with* $\tau = 50$*; dynamic momentum term is* $\frac{k}{k+3}$*, where* k *is the number of iterations*

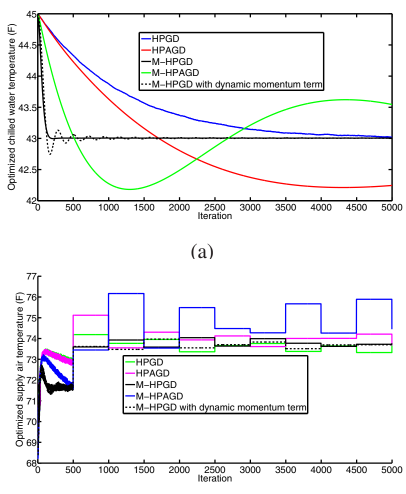


Fig. 7. *Optimized (a) chilled water temperature set point and (b) supply air temperature set point using different methods with* $\tau = 500$*; dynamic momentum term is* $\frac{k}{k+3}$*, where* k *is the number of iterations*

sumption optimization and effect of communication period. Beyond the existing work, several future work directions can include: 1) using real test bed system data and controller in the hierarchical optimization framework and 2) showing theoretical convergence analysis for the proposed algorithm;

REFERENCES

[1] C. Blasco, J. Monreal, I. Benítez, and A. Lluna, "Modelling and pid control of hvac system according to energy efficiency and comfort criteria," in *Sustainability in Energy and Buildings*, pp. 365–374, Springer, 2012.

[2] K. Mařík, J. Rojíček, P. Stluka, and J. Vass, "Advanced hvac control: Theory vs. reality," *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 3108–3113, 2011.

[3] V. Chinde, A. Kohl, Z. Jiang, A. Kelkar, and S. Sarkar, "A volttron based implementation of supervisory control using generalized gossip for building energy systems," 2016.

[4] Z. Jiang, V. Chinde, A. Kohl, S. Sarkar, and A. Kelkar, "Scalable supervisory control of building energy systems using generalized gossip," in *American Control Conference (ACC), 2016*, pp. 581–586, IEEE, 2016.

[5] G.-Y. Jin, W.-J. Cai, L. Lu, E. L. Lee, and A. Chiang, "A simplified modeling of mechanical cooling tower for control and optimization of hvac systems," *Energy conversion and management*, vol. 48, no. 2, pp. 355–365, 2007.

[6] A. Hauswirth, S. Bolognani, G. Hug, and F. Dörfler, "Projected gradient descent on riemannian manifolds with applications to online power system optimization.," in *Allerton*, pp. 225–232, 2016.

[7] C. Liu, C. Lee, H. Chen, and S. Mehrotra, "Stochastic robust mathematical programming model for power system optimization," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 821–822, 2016.

[8] A. F. Taha, N. Gatsis, B. Dong, A. Pipri, and Z. Li, "Buildings-to-grid integration framework," *arXiv preprint arXiv:1706.05626*, 2017.

[9] L. Lu, W. Cai, Y. S. Chai, and L. Xie, "Global optimization for overall hvac systems—-part i problem formulation and analysis," *Energy conversion and management*, vol. 46, no. 7, pp. 999–1014, 2005.

[10] Z. Yu, Y. Zhou, and A. Dexter, "Hierarchical fuzzy rule-based control of renewable energy building systems," *CISBAT, Renewables in a changing climate-Innovation in the Built Environment*, 2007.

[11] M. Maasoumy, A. Pinto, and A. Sangiovanni-Vincentelli, "Model-based hierarchical optimal control design for hvac systems," in *Dynamic System Control Conference (DSCC)*, 2011.

[12] K. Yun, H. Cho, R. Luck, and P. Mago, "Real-time combined heat and power operational strategy using a hierarchical optimization algorithm," *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, vol. 225, no. 4, pp. 403–412, 2011.

[13] R. Choudhary, P. Papalambros, and A. M. Malkawi, "Simulation-based design by hierarchical optimization," in *Proceedings of Ninth International IBPSA Conference.–Montreal, Canada*, pp. 175–182, 2005.

[14] A. Lefort, R. Bourdais, G. Ansanay-Alex, and H. Guéguen, "Hierarchical control method applied to energy management of a residential house," *Energy and Buildings*, vol. 64, pp. 53–61, 2013.

[15] E. Vrettos, E. C. Kara, J. MacDonald, G. Andersson, and D. S. Callaway, "Experimental demonstration of frequency regulation by commercial buildings–part i: Modeling and hierarchical control design," *IEEE Transactions on Smart Grid*, 2016.

[16] N. Elbaccouche, B. Boussaid, N. Abdelkrim, and C. Aubrun, "Hierarchical predictive control for multizone building," in *Control, Automation, Robotics and Vision (ICARCV), 2016 14th International Conference on*, pp. 1–7, IEEE, 2016.

[17] L. Zhuang, X. Chen, and X. Guan, "A bi-level optimization for an hvac system," *Cluster Computing*, pp. 1–13, 2017.

[18] G. Londono and A. Lozano, "A bilevel optimization program with equilibrium constraints for an urban network dependent on time," *Transportation Research Procedia*, vol. 3, pp. 905–914, 2014.

[19] B. Colson, P. Marcotte, and G. Savard, "An overview of bilevel optimization," *Annals of operations research*, vol. 153, no. 1, pp. 235–256, 2007.

[20] N. Couellan and W. Wang, "On the convergence of stochastic bi-level gradient methods," *Optimization*, 2016.

[21] A. Sinha, P. Malo, and K. Deb, "A review on bilevel optimization: From classical to evolutionary approaches and applications," *IEEE Transactions on Evolutionary Computation*, 2017.

[22] O. L. Mangasarian and S. Fromovitz, "The fritz john necessary optimality conditions in the presence of equality and inequality constraints," *Journal of Mathematical Analysis and applications*, vol. 17, no. 1, pp. 37–47, 1967.

[23] D. Zhang and G.-H. Lin, "Bilevel direct search method for leader-follower equilibrium problems and applications," *Centre for Health Services Research, Singapore Health Services; School of Mathematical Sciences, Dalian University of Technology*, pp. 1–31, 2012.